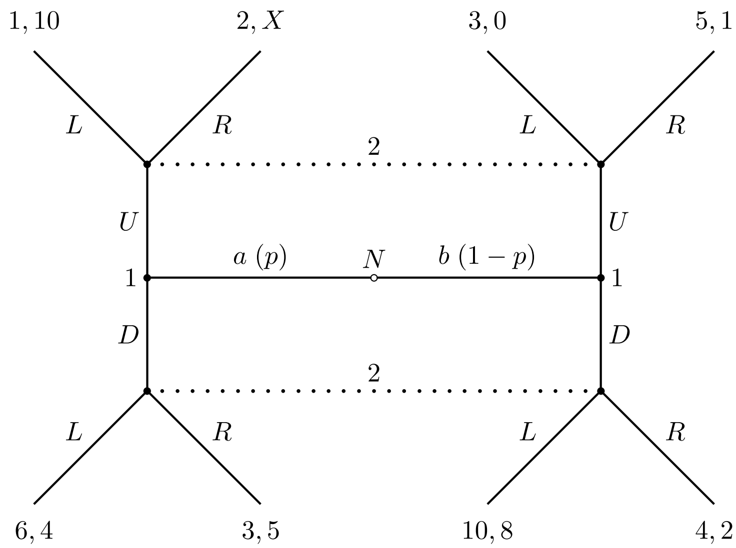
Question 1. Consider the following signaling game. Nature selects between type a (with probability p) and b (with probability $1-p$). The variables p and X are parameters. Player 1's payoffs are given first.



(a) Assume that $p = 1/2$ and $X = 5$. Find all *separating* perfect Bayesian equilibria (if any).

(b) Would your answer to part (a) change for different values of X or p? Carefully explain why or why not?

(c) Does any separating equilibrium from part (a) satisfy the intuitive criterion? Carefully explain.

(d) Assume that $p = 1/2$ and $X = 5$. Find all pure-strategy *pooling* perfect Bayesian equilibria (if any).

(e) Find *all* values of p and X for which this game has a pure strategy pooling equilibrium. Carefully explain.

Question 2. Consider a principal-agent problem. An agent puts in one of two effort levels, l or h, at a cost of $c(l) = 0$ and $c(h) = 10$. There are two possible outcomes, $\{x_1, x_2\} \equiv \{200, 1000\}$. The probability of x_2 is 1/4 if the agent puts in low effort and is 3/4 if the agent puts in high effort.

The principal is risk neutral with utility function $x - w$ where x is the realized outcome and w is the wage paid to the agent. The agent has utility function $U(w, e) = w^k - c(e)$, $0 < k < 1$. The agent's reservation utility is 0. The principal selects a pair of wages, w_1 and w_2, corresponding to each outcome.

Wages are restricted to be non-negative.

(a) What is the optimal incentive contract for inducing low effort?

(b) What is the optimal incentive contract for inducing high effort?

[Carefully consider which constraints bind]

(c) Write down a condition for when inducing high effort is optimal? Intuitively explain this condition.

(d) If effort were observable so that the principal could directly contract on effort level, would the range of k under which it would be optimal to induce high effort be larger or smaller than that obtained in (c)? Carefully and intuitively explain why.

(e) Imagine that outcome x_2 now becomes two separately identifiable outcomes, x_{2a} and x_{2b}. At each effort level, the probabilities of x_1 remain unchanged but 2/3 of the probability of x_2 flows to x_{2b}. The probabilities of each outcome are now given by:

	x_1	x_{2a}	x_{2b}
low effort:	3/4	1/12	2/12
high effort:	1/4	3/12	6/12

What is the optimal incentive contract for inducing high effort?

Question 3. Consider a simultaneous-move game between two players, $i \in \{1, 2\}$, where player i selects $x_i \in \Re$. When the two players select x_i and x_j, player i receives payoffs of

$$u(x_i, x_j) = -(x_i - A_i)^2 - (x_i - x_j)^2$$

That is, each player suffers disutility the further his strategy is from $A_i \geq 0$ and the further his strategy is from the other player.

(a) Assume that $A_1 = 10$ and $A_2 = 40$. Find the Nash equilibrium.

(b) Consider a two-stage game. In stage 1, players simultaneously select A_i. In stage 2, players observe stage 1 choices and then simultaneously select x_i. Find all pure-strategy subgame perfect Nash equilibria.