Microeconomic Theory II   Spring 2014 Final Exam - ROUGH solutions

These are not complete, and may contain errors.

Question 1. Note that Player 1 of type a will always select D as it yields a payoff of at least 3, which is higher than either payoff from U (this is different from saying that it is "dominated"!). Therefore, the only strategy for player 1 that can form the basis of a separating equilibrium is D (for type a) and U (for type b). Player 2's best response to $b \to U$ or $a \to D$ is R. We can confirm that player 1 would not deviate, so $\{D, D; R, R\}$ with degenerate beliefs forms the only separating equilibrium.

This answer does not depend on p as it is a separating equilibrium, and does not depend on X because type a plays D regardless of what Player 2 may do.

This equilibrium satisfies the intuitive criterion because there is no unsent message. Note that this is NOT a general property of separating equilibria unless there are as many messages as there are types.

To find pooling equilibria, we only need to consider D, D for the same reason as above. Player 2's best response to D is L, which yields both types for player 1 higher payoffs than they could obtain from U regardless of what player 2 does. Therefore, anything in which both types pick D, player 2 picks L in response to D, player 2 has any off-equilibrium beliefs, and player 2 plays a best response to those beliefs in case of U, is a pooling equilibrium.

To find the values of X and p for which pooling equilibria exist, we need player 2's best response to D to be L, or if it is R, then the best response to U must be L (to keep type b from deviating). This implies that either $p \leq 6/7$ or, when $p > 6/7$, $X \leq 10$.

Question 2.

(a) For low effort, we require $w_1 = w_2 = 0$.

(b) For high (unobservable) effort, we have four constraints:
IC: $w_1^k + w_2^k \geq 20 \equiv w_2^k \geq 20 - w_1^k$.

IR: $\frac{1}{4}w_1^k + \frac{3}{4}w_2^k \geq 10 \; equiv w_2^k \geq \frac{40}{3} - \frac{1}{3}w_1^k$

Non-negativity for 1: $w_1 \geq 0$

Non-negativity for 2: $w_2 \geq 0$

The IC constraint and the non-negativity for 1 constraint imply that the other two constraints don't bind. The other two must. Thus, $w_1 = 0$ and $w_2 = 20^{\frac{1}{k}}$.

(c) Compare the profits for high and low effort (evaluated at the optimal wages above) and this yields a condition on k. High effort is desirable when k is sufficiently large, which implies that the agent is not too risk averse.

(d) This question doesn't ask for the range, just to compare it to (c). When effort is observable, profit for high effort is higher, but profit for low effort remains unchanged. Thus, high effort will be preferred over a broader range of k. This is because there is not a risk premium to pay.

(e) This question sent many of you down a long computational path unnecessarily. Imagine if the set-up of the original problem were changed slightly so that when outcome 2 is obtained, a die is rolled that is not observed by the agent or principal, and both the agent and principal are informed that "outcome 2a" happened if the die lands on 1 or 2, and that "outcome 2b" happened otherwise. How would the optimal compensation change? Clearly, the random die does not provide any extra information about the effort level. So, why would wages vary with the outcome of this die? This question is equivalent.

Recall that we derived in class how the optimal wage in each state depends on the relative probability of achieving that state under low and high effort levels. Confirm that outcomes 2a and 2b have the same relative probabilities and these are the same as outcome 2 in the original problem. Put simply, nothing changes from part (a) to part (e).

Question 3. Each player's best response is given by $x_i = \frac{1}{2}(A_i + x_j), j = 3 - i$. Intuitively, since utility is decreasing in distance from A_i and x_j symmetrically, the best response is to locate in the middle.

This implies equilibrium locations of $x_i = \frac{2}{3}A_i + \frac{1}{3}A_j$.

In part (a), substituting 10 and 40 for A_1 and A_2 yields an equilibrium of $\{20, 30\}$.

For part (b), we substitute the stage 2 equilibrium into the utility function, which yields a utility function proportional to $(A_1 - A_2)^2$. Each player's best response is then $A_i(A_j) = A_j$, implying that $A_1 = A_2 = A$ is an equilibrium of the first stage for any A.