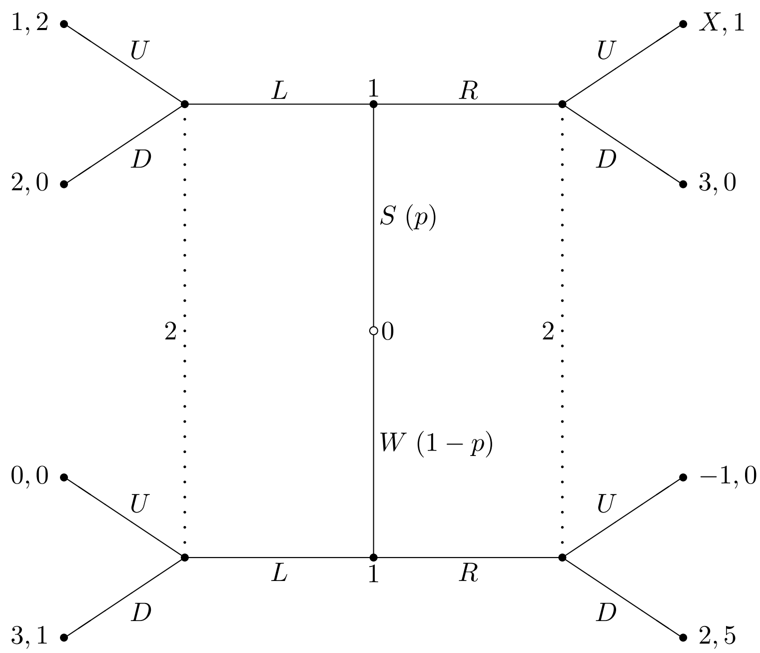
Microeconomic Theory II — Spring 2015

Final Exam SOLUTIONS — Mikhael Shor

Short, incomplete, and not guaranteed.

Question 1. Consider the following game. First, nature (player 0) selects S with probability p, $0 < p < 1$, or W with probability $1 - p$. Next, player 1 selects L or R. Lastly, player 2 selects U or D. The game has two parameters: X and p.



(a) For what values of the parameters p and X does the above game have a pooling equilibrium that includes $\{R, R\}$? If the receiver were to pick U in response to R, type W would select L. Therefore, $R \to D$ requiring $p \leq \frac{5}{6}$. Type S would not deviate, and type W would not deviate if $L \to U$ which is feasible for some beliefs. Therefore $p \leq \frac{5}{6}$ and any X.

(b) Does the pooling equilibrium above satisfy the intuitive criterion? It does not. Type S would not deviate for any beliefs of Player 2 ($3 > 2$ and $3 > 1$). If type W were to deviate, Player 2 (knowing it is type W), responds with D, making the deviation profitable for type W. Therefore, all conditions of the intuitive criterion are satisfied.

(c) For what values of the parameters does the above game have a separating equilibrium? What is the equilibrium? First check L, R: $L \to U, R \to D$. But then type S prefers R (earning 3) to L (earning 1) so this cannot be an equilibrium. Next, check R, L: $L \to D, R \to U$. Type W would not deviate $(3 > -1)$ and type R would not deviate if $X \geq 2$. Therefore, $X \geq 2$ and any p.

(d) For what values of the parameters does the above game have a pooling equilibrium that includes $\{L, L\}$? Carefully demonstrate or explain. First note that the equilibrium cannot involve $R \to D$ as type S would always select R. Therefore, the equilibrium must involve $R \to U$. Then, there are two possibilities.

First, $L \to U$ (which requires $p \geq \frac{1}{3}$). Type W is playing a best response, and type S is if $X \leq 1$.

Second, $L \to D$ (which requires $p \leq \frac{1}{3}$). Type W is playing a best response, and type S is if $X \leq 2$.

Combining: Either $X \leq 1$ or both $p \leq \frac{1}{3}$ and $X \leq 2$.

Question 2. Consider a principal-agent model in which the agent has three levels of effort (low, medium, or high) and there are two possible outcomes (associated with profits for the principal of 180,000 and 500,000). The principal is risk neutral with utility given by profits minus wages. The agent's utility function is (of course) given by $u(w, e) = \sqrt{w} - c(e)$, and the reservation utility is 0. Wages cannot be negative. The relevant data are:

	profit		
effort level	500,000	180,000	c(e)
low	$\frac{2}{8}$	$\frac{6}{8}$	100
med	$\frac{3}{8}$	$\frac{5}{8}$	250
high	$\frac{4}{8}$	$\frac{4}{8}$	c_h

where c_h is the cost of high effort.

For what values of c_h is the principal's optimal profit the same whether or not effort is observable? Carefully explain.

Given that the agent is risk averse, the principal's optimal profit under perfect information is obtained by paying a constant wage. Since medium and high effort cannot be induced with a constant wage when effort is not observable, their profit must be lower than when effort is observable. Thus, the only way that the two profits are equal is when low effort is optimal under observable effort (which implies that it is optimal under unobservable effort). This requires that low effort is more profitable than medium effort, high effort, and no effort:

$$\frac{2}{8}500,000 + \frac{6}{8}180,000 - 100^2 \geq \frac{3}{8}500,000 + \frac{5}{8}180,000 - 250^2$$

and

$$\frac{2}{8}500,000 + \frac{6}{8}180,000 - 100^2 \geq \frac{4}{8}500,000 + \frac{4}{8}180,000 - c_h^2$$

and

$$\frac{2}{8}500,000 + \frac{6}{8}180,000 - 100^2 \geq 0$$

which reduces to $c_h \geq 300$.

Question 3. Consider a duopoly (two firm) market with inverse demand given by $P = 80 - Q = 80 - q_1 - q_2$. Each firm has marginal cost of $c = 20$.

The firms play the following two-stage game. First, the owners of the firms (who aim to maximize profits) simulatenously select managers, m_1 and m_2, where $m_i \in [0, 1]$. Second, the managers simultaneously select outputs, q_i. Manager m_i selects q_i to maximize $m_i\pi_i + (1 - m_i)R_i$ where π_i is the profit and R_i is the revenue of firm i.

1. Find the subgame perfect Nash equilibrium of this game.
   Note that the second stage is just a standard linear Cournot duopoly with each manager effectively considering marginal cost to be $20m_i$. Maximizing profit gives the best response function $q_i = 40 - 10m_i - \frac{1}{2}q_j$. Solving the two simultaneously yields $q_i = \frac{1}{3}(80 - 40m_i + 20m_j)$.

   In the first stage, the owner is just maximizing profit $p_i(q_i, q_j)q_i - 20q_i$ but we substitute in the second-stage equilibrium to express profit only as a function of m_1 and m_2:

   $$\Pi_1 = \left(\frac{20}{3}\right)^2 (1 + m1 + m2)(4 - 2m_1 + m_2)$$

   Maximizing yields the best response function $m_1 = \frac{2-m_2}{4}$ and similarly for player 2. This yields the SPNE: $m_1 = m_2 = \frac{2}{5}$ and $q_i = \frac{1}{3}(80 - 40m_i + 20m_j)$.

2. Why is the owner asking the manager to put some weight on revenue rather than profit? Does this lead to higher or lower overall industry profits? Briefly, intuitively explain. Placing weight on revenue reduces the effective marginal cost and increases output. It also has the effect of decreasing the rival's output, so it acts like a Stackelberg leader. However, since both firms do this, total output increases, reducing industry profit.