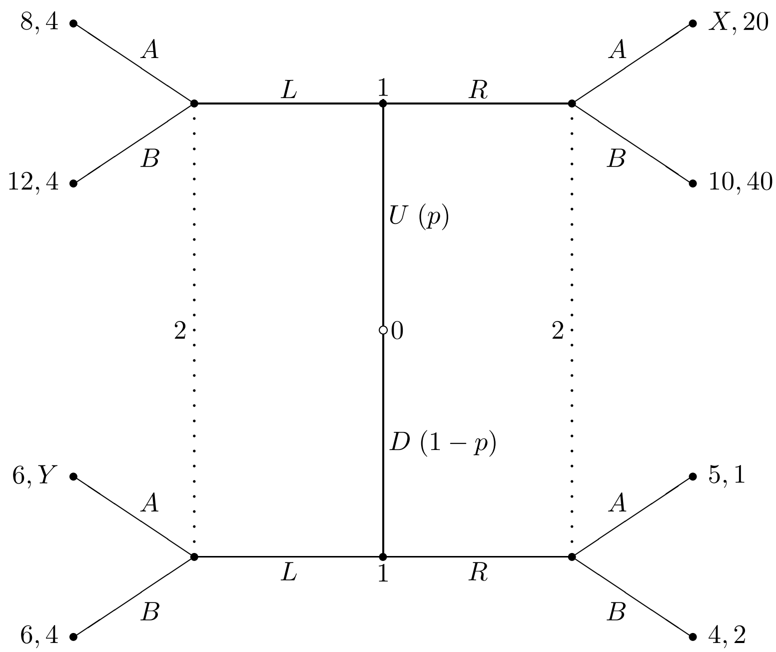
Microeconomic Theory II — Spring 2017
Final Exam Solutions — Mikhael Shor

Warning: Very brief, incomplete, and quite possibly incorrect.

Question 1. Consider the following game. First, nature (player 0) selects U with probability p, $0 < p < 1$, or D with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects A or B. The game has three parameters: X, Y, and p.



(a) For what values of the parameters does the above game have a separating equilibrium? Explain.

In response to R, player 2 will select B for *any* beliefs.[1] Therefore, the value of X cannot matter. Also note that type D would never choose R (payoffs of 4 or 5 versus 6) so there is only one potential separating equilibrium.

(i) $D \to L, U \to R$

(ii) $\mu(D|L) = \mu(U|R) = 1$

(iii) $L \to A$ or B (depending on whether $Y \geq 4$ or $Y \leq 4$); $R \to B$

(iv) $U \to R$ requires $L \to A$; $D \to L$ always

[1] Some students stated that B is "dominant." Note that B is an action, not a strategy.

Therefore, to have a separating equilibrium, we need $L \to A$. This is true whenever $Y \geq 4$, for any p and X.

(b) Do these separating equilibria satisfy the intuitive criterion? Explain.

There is no unsent message. Therefore, all separating equilibria in this game satisfy the intuitive criterion.

(c) For what values of the parameters does the above game have a pooling equilibrium? Explain.

Again, since type D will always choose L, we only need to consider one possibility.

(i) $D \to L, U \to L$

(ii) $\mu(D|L) = 1 - p$, $\mu(D|R) = ?$

(iii) $L \to A$ or B (depending on whether $Y \geq 4$ or $Y \leq 4$); $R \to B$ for any beliefs

(iv) $U \to L$ requires $L \to B$; $D \to L$ always

Therefore, we need $L \to B$ which requires $Y \leq 4$, for any p or X.

(d) Do these pooling equilibria satisfy the intuitive criterion? Explain.

Type D would never deviate since 6 is greater than any payoff from R. If type U were to deviate, player 2 would play B, which would yield lower than equilibrium payoffs for type U. Thus, type U will not deviate. The Intuitive Criterion is satisfied.

(e) Explain why *neither* of the answers to (a) or (c) about pooling or separating equilibria depends on the value of p. Discuss if this is a general result of all signaling games or specific to this game.

Separating equilibria never depend on p is beliefs are degenerate (the type is always known). Pooling equilibria generally depend on p (since posterior beliefs along the equilibrium path are equal to the prior beliefs). Here, it does not matter because, in response to L, the payoffs in the top left are the same for player 2, so only $Y \gtreqless 4$ matters.

Question 2. Consider a principal-agent model in which the agent has three levels of effort (low, medium, or high) and there are three possible outcomes (associated with profits for the principal of 20,000, 40,000, or 100,000). The principal is risk neutral with utility given by profits minus wages. The agent's utility function is given by $u(w, e) = \sqrt{w} - c(e)$, and the reservation utility is 0.

The following table provides the probability of each outcome given a level of effort and the agent's cost of effort.

By law, the agent's wage cannot be negative.

	profit			
effort level	20,000	40,000	100,000	c(e)
low	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	10
med	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	65
high	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	110

(a) Assume that the principal can observe effort. What is the optimal contract? Show or explain.

Note that there are (in theory) nine wages we can set - one for each combination of outcomes and effort levels. However, since the agent is risk averse, we know that wages will not vary with outcome.

First, we derive the optimal contract *for each effort level* which is obtained when $\sqrt{w^e_{20k}} = \sqrt{w^e_{40k}} = \sqrt{w^e_{100k}} = c(e)$.

Induce low: $w^L_{20k} = w^L_{40k} = w^L_{100k} = 100$

Induce med: $w^M_{20k} = w^M_{40k} = w^M_{100k} = 4225$

Induce high: $w^H_{20k} = w^H_{40k} = w^H_{100k} = 12100$

Second, we derive the profitability of each:

$\pi^L = \frac{1}{4}20,000 + \frac{1}{2}40,000 + \frac{1}{4}100,000 - 100 = 49,900$

$\pi^M = \frac{1}{4}20,000 + \frac{3}{8}40,000 + \frac{3}{8}100,000 - 4,225 = 53,275$

$\pi^H = \frac{1}{4}20,000 + \frac{1}{4}40,000 + \frac{1}{2}100,000 - 12,100 = 52,900$

Therefore, we wish to contract on medium effort. This is achieved by setting $w^M_{20k} = w^M_{40k} = w^M_{100k} = 4225$ and by setting w^L and w^H sufficiently low to deter other effort levels (e.g., 0).

(b) Assume that the principal cannot observe effort (but can observe the outcome). What is the optimal contract? Carefully explain how you obtain your answer.

[Minor hints: *IF* you really feel the need to set up an optimization problem (i) at least one of the non-negative wage constraints will be binding and (ii) a derivative is not necessary.]

First, low effort is induced as in the perfect information case, and therefore yields profits of 0.

Second, medium effort is not implementable since the IC constraints $M \succsim L$ and $M \succsim H$ yield a contradiction.

For high effort, one could solve the constrained optimization problem (which would yield $w_{20k} = w_{40k} = 0; w_{100k} = 160000$. But, there is a much easier way. Note that one of the IC constraints,

$$\frac{1}{4}\sqrt{w_{20k}} + \frac{1}{4}\sqrt{w_{40k}} + \frac{1}{2}\sqrt{w_{100k}} - 110 \geq \frac{1}{4}\sqrt{w_{20k}} + \frac{1}{2}\sqrt{w_{40k}} + \frac{1}{4}\sqrt{w_{100k}} - 10$$

reduces to $\sqrt{w_{100k}} \geq \sqrt{w_{40k}} + 400$

Along with the minimum wage constraint $w_{40k} \geq 0$, we know that inducing high effort requires wages of *at least* $400^2 = 160,000$ when the outcome $100k$ obtains. But this fact (combined with the fact that all other wages are non-negative) implies that profit from inducing high effort is negative.

Therefore, implementing low effort is optimal. The optimal contract is $\sqrt{w_{20k}} = \sqrt{w_{40k}} = \sqrt{w_{100k}} = 100$.

Question 3. Three widget manufacturers are planning to build factories in Storrs. Each firm ($i = 1, 2, 3$) must decide on the capacity (c_i) of its factory. These decisions are made in the following order:

1. First, firms 1 and 2 simultaneously select their capacity levels, c_1 and c_2.
2. Second, firm 3 observes c_1 and c_2 and selects a capacity c_3.

Once built, all firms operate at capacity. The industry price is given by:

$$p = 2 - \sum_{i=1}^{3} c_i$$

Building capacity of c has a cost of c. Therefore, profits for firm i are given by:

$$pc_i - c_i$$

Determine the subgame perfect equilibrium.

This is a Stackelberg problem with "capacities" replacing "quantities." We solve the problem backwards, starting with period 2.

Stage 2: Maximizing $\pi_3 = [2-(c_1+c_2+c_3)]c_3 - c_3$ yields $c_3^* = \frac{1}{2}(1-c_1-c_2)$.

Stage 1: The first stage is a Cournot game, but we must recognize that $c_3(c_1, c_2)$ is a function of first stage decisions.

$$\begin{aligned}
\pi_1 &= [2 - (c_1 + c_2 + c_3(c_1, c_2))]c_1 - c_1 \\
&= [2 - (c_1 + c_2 + \frac{1}{2}(1 - c_1 - c_2)]c_1 - c_1 \\
&= [\frac{3}{2} - \frac{1}{2}(c_1 + c_2)]c_1 - c_1 \\
&= \frac{1}{2}c_1 - \frac{1}{2}(c_1 + c_2)c_1
\end{aligned}$$

Maximizing yields firm 1's best response: $c_1(c_2) = \frac{1}{2}(1 - c_2)$.
Similarly, $c_2(c_1) = \frac{1}{2}(1 - c_1)$.
Solving simultaneously yields $c_1^* = c_2^* = \frac{1}{3}$.
Therefore, the subgame perfect Nash equilibrium is

$$\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{2}(1 - c_1 - c_2)\right\}$$