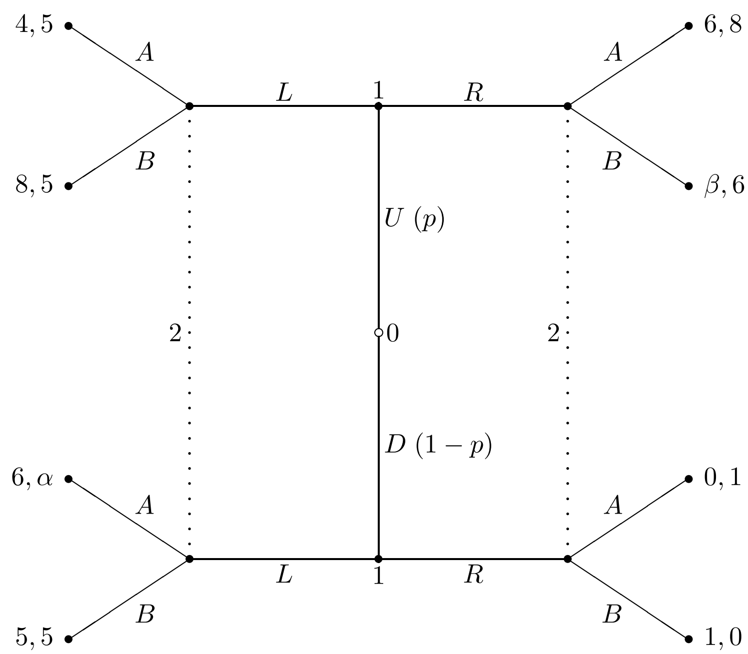
Microeconomic Theory II Final Exam SOLUTIONS	Spring 2019 Mikhael Shor

YOUR GRADE IS ON THE LAST PAGE OF YOUR EXAM
Each question was graded out of 40 points, for a total of 120.
Note: these are not meant to be complete solutions and are not guaranteed to be accurate.

Question 1. Consider the following game. First, nature (player 0) selects U with probability p, $0 < p < 1$, or D with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects A or B. The game has three parameters: α, β, and p.



(a) Assume $\alpha = \beta = 0, p = \frac{1}{2}$. Derive all pure-strategy perfect Bayesian equilibria.

(b) For what values of the parameters does the above game have a separating equilibrium? Does this separating equilibrium satisfy the intuitive criterion?

(c) For what values of the parameters does the above game have a pooling equilibrium? Explain.

(d) Explain why *neither* of the above two answers (b and c) depends on the value of p. Carefully discuss if this is a general result or specific to this game.

Solution: Start with (b) and (c) since (a) will be a special case of (one of) those. First, note that type D will always pick L (both 6 and 5 are greater than either 0 or 1). Second, note that, for any beliefs following a message of R, player 2 will always select A ($8q + 1 - q > 6q$).

(b) We need only consider $U \to R, D \to L$. Since $D \to L$ is always a best response, we need only ensure that $U \to R$ is a best response. Since $U \to R \to A$ yields player 1 a payoff of 6, we need $L \to A$ (since $8 > 6 > 4$) which is a best response (to $D \to L$) only if $\alpha \geq 5$. This separating equilibrium satisfies the intuitive criterion as there is no unsent message in equilibrium.

(c) We need only consider pooling on L. Since $D \to L$ is always a best response, we need only ensure that $U \to L$ is a best response. Since $U \to R \to A$ yields player 1 a payoff of 6, we need $L \to B$ (since $8 > 6 > 4$) which is a best response (to beliefs $p, 1-p$) only if $\alpha \leq 5$.

(a) Since $\alpha = 0 < \frac{1}{2}$, we know that we have only a pooling equilibrium: $\{U \to L, D \to L; L \to B, R \to A\}$ with beliefs $\mu(U|L) = p = \frac{1}{2}, 0 \leq \mu(U|R) \leq 1$.

(d) Separating equilibria never depend on p since on-equilibrium beliefs are degenerate. For pooling equilibria, p directly impacts on-equilibrium beliefs. In the special case of this game, however, best responses to L and R are independent of beliefs.

Question 2. Consider a principal-agent model in which the agent has three levels of effort (low, medium, or high) and there are two possible outcomes (associated with revenues for the principal of either 0 or 90). The principal is risk neutral with profit given by revenues minus wages. The agent's utility is given by $u(w, e) = \sqrt{w} - c(e)$. The reservation utility is 0.

The probability of the good outcome (revenues of 90) is given by $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, for low, medium, and high effort, respectively.
The cost of effort, $c(e)$, is 4, $4\frac{4}{5}$, 5, for low, medium, and high effort, respectively. By law, the agent's wage cannot be negative.

(a) Assume that the principal can observe the agent's effort. What is the principal's optimal (profit maximizing) contract? Demonstrate or explain.

(b) Assume that the principal cannot observe the agent's effort and the contract can depend only on realized revenue. What is the principal's optimal (profit maximizing) contract? Demonstrate or explain.

Solution:
(a) If effort is observable then:

for effort level...	we pay wage...	and earn profit...
low	$4^2 = 16$	$\frac{1}{3}90 - 16 = 14$
medium	$(4\frac{4}{5})^2$	$\frac{1}{2}90 - (4\frac{4}{5})^2 < 30$
high	$5^2 = 25$	$\frac{2}{3}90 - 25 = 35$

Thus, highest profit is obtainable by paying $w_H = 25$ and paying sufficiently low wages for low and medium effort.

(b) If effort is unobservable, then low effort is obtainable as above. Medium effort is not implementable (as the IC conditions for $M \succ L$ and $M \succ H$ contradict). For high effort, we have:

IR: $\frac{2}{3}\sqrt{w_{90}} + \frac{1}{3}\sqrt{w_0} - 5 \geq 0$

IC: $\frac{2}{3}\sqrt{w_{90}} + \frac{1}{3}\sqrt{w_0} - 5 \geq \frac{1}{3}\sqrt{w_{90}} + \frac{2}{3}\sqrt{w_0} - 4$

Note that the other IC condition ($H \succ M$) is implied by the above, and the non-negativity IR conditions won't bind. Since both the IR and the IC constraints must bind, we obtain $w_0 = 9, w_{90} = 36$. The resulting profit is $\frac{2}{3}90 - \frac{2}{3}36 - \frac{1}{3}9 = 33$. Since this is higher than profit from low effort (and medium effort is not possible), the optimal contract is $w_0 = 9, w_{90} = 36$.

Question 3. The world's diamonds are controlled by two cartels, De Beers and ALROSA. Each year, at the annual diamond wholesaler convention, both cartels announce their mining targets for the year–the quantity of diamonds that they commit to mining–and the wholesalers commit to a wholesale price, w, as a function of these commitments.

This year, the convention has two time slots available for announcements: Tuesday at noon and Wednesday at noon. Both cartels can choose the same time slot (and announce in different rooms) or different time slots.

The timing of the game is as follows:

(a) Both De Beers and ALROSA, $i \in \{D, A\}$, simultaneously select whether to announce on Tuesday or on Wednesday, $a_i \in \{T, W\}$. Then, the conference program is published and made publicly available.

(b) Firms that chose to announce on Tuesday (if any) announce $m_i \in \Re$, the annual mining target. The minutes of Tuesday's meeting (including all announced mining targets) are published and made publicly available.

(c) Firms that chose to announce on Wednesday (if any) announce m_i.

(d) The diamond wholesalers announce a wholesale price given by $w = 1 - m_A - m_D$

(e) De Beers and ALROSA realize profits of $\Pi_i = w\, m_i$

Determine the pure strategy subgame perfect equilibria.

Solution: If, in the first period, both select the same day (both Tuesday or both Wednesday), then the resulting game is Cournot with equilibrium strategies $m_A = m_D = \frac{1}{3}$ and profits $\pi_A = \pi_D = \frac{1}{9}$.

If in the first period, both select different days (e.g., A picks T and D picks W), then the resulting game is Stackelberg with equilibrium strategies $m_A = \frac{1}{2}$, $m_D = \frac{1}{2}(1 - m_A)$ and equilibrium profits $\pi_A = \frac{1}{8}$, $\pi_D = \frac{1}{16}$.

Then, in the first period, the resulting game is:

		Player 2	
		T	W
Player 1	T	$\frac{1}{9}, \frac{1}{9}$	$\frac{1}{8}, \frac{1}{16}$
	W	$\frac{1}{16}, \frac{1}{8}$	$\frac{1}{9}, \frac{1}{9}$

Since $\frac{1}{8} > \frac{1}{9}$ and $\frac{1}{9} > \frac{1}{16}$, each player has a dominant strategy to select T. The unique subgame perfect equilibrium is the actions T, T in the first period and the actions of each player in each of the four subgames.