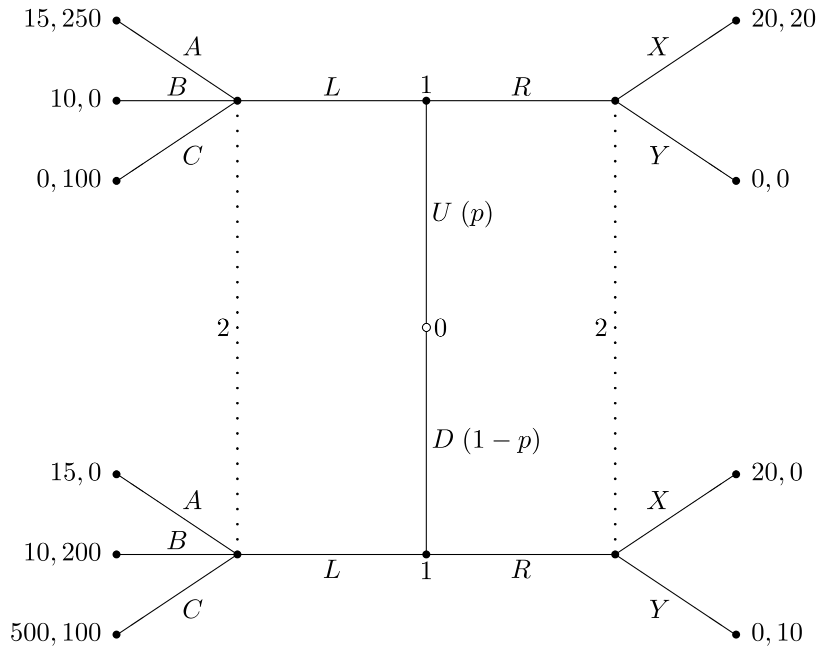
Microeconomic Theory II — Spring 2022

Final Exam — Mikhael Shor

Carefully explain and support your answers.

Question 1. Consider the following game. First, nature (player 0) selects U with probability p or D with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects either A, B, or C (if player 1 selected L) or X or Y (if player 1 selected R).



(a) What are each player's pure strategies?

(b) Assume $p = \frac{1}{2}$. Find all pure-strategy weak perfect Bayesian equilibria (and show that none other exist).

(c) For each equilibrium found above, show whether or not it satisfies the Intuitive Criterion.

(d) For what values of p does this game have a pooling equilibrium? Demonstrate or explain.

Question 2. Consider a principal-agent model in which the agent has three levels of effort, $e \in \{L, M, H\}$. There are three different outcomes associated with different profits for the principal, (π_1, π_2, π_3). Define p_i^e as the probability of outcome i when level of effort is e.

The principal is risk neutral with utility given by profits minus wages. The agents utility function is (of course) given by $u(w, e) = \sqrt{w} - c(e)$.

The cost to the agent of the three types of effort are $c(L) = 0, c(M) = 200, c(H) = 500$. Reservation utility is 0.

		outcome 1	outcome 2	outcome 3
(π_1, π_2, π_3)	$=$	$1,000,000$	$4,000,000$	$8,000,000$
(p_1^L, p_2^L, p_3^L)	$=$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
(p_1^M, p_2^M, p_3^M)	$=$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
(p_1^H, p_2^H, p_3^H)	$=$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Wages cannot be negative.

(a) If effort can be observed, what is the optimal contract? Demonstrate.

(b) Assume that effort cannot be observed (but outcomes can). Derive the optimal contract for *each* level of effort. Show all constraints.

(c) If effort cannot be observed, what is the optimal contract?

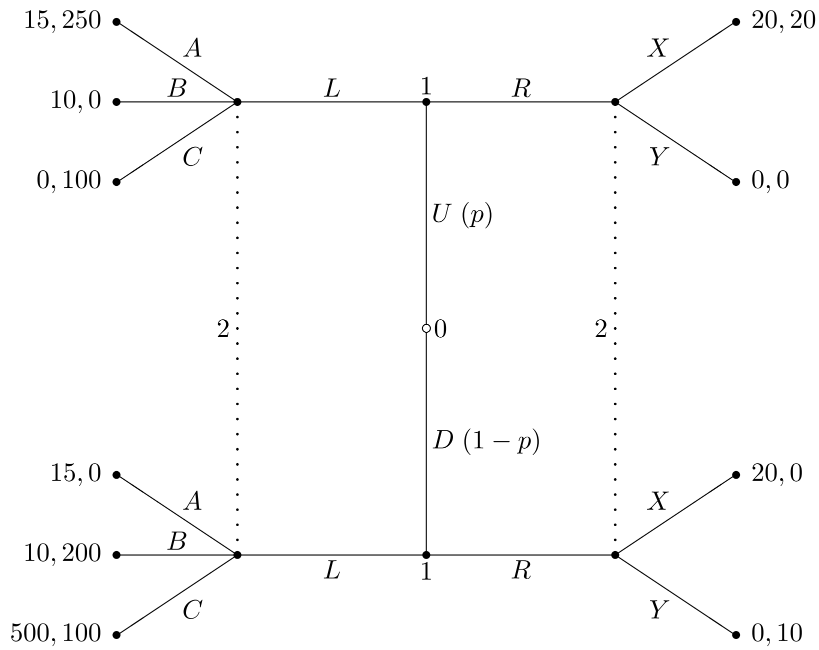
Question 3. Consider a differentiated-products version of a Bertrand duopoly (firms i and j). Firm $i \neq j$ has demand given by

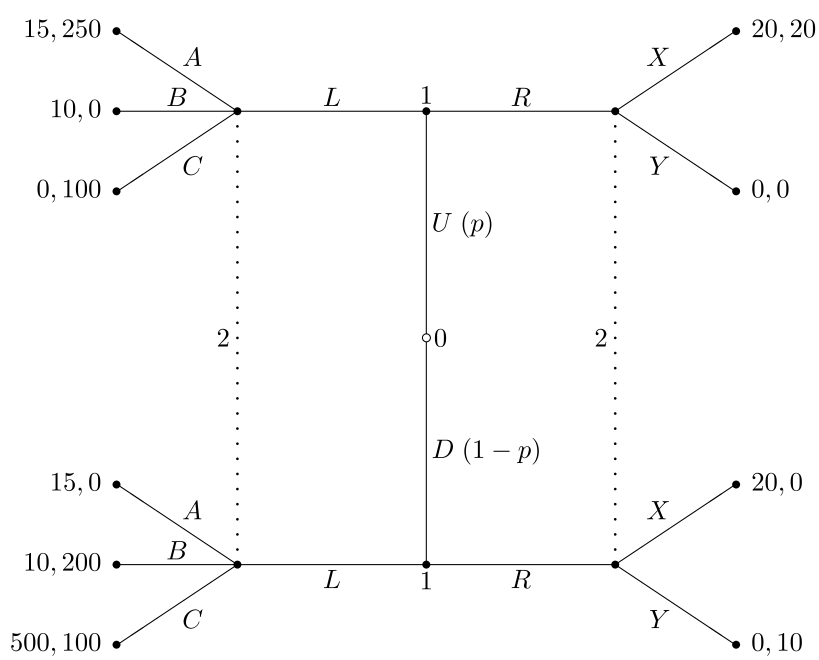
$$q_i = 168 - 2p_i + p_j$$

with no costs of production. Each firm's profit is $p_i q_i$.

Determine firm i's subgame-perfect equilibrium profit if:

1. The firms choose p_i and p_j simultaneously.
2. The firms choose p_i and p_j sequentially, with firm i choosing first.
3. The firms choose p_i and p_j sequentially, with firm j choosing first.


15, 250
A
10, 0
B
L
1
R
X
20, 20
C
0, 100
Y
0, 0
U (p)
2
0
2
D (1 − p)
15, 0
A
20, 0
X
10, 200
B
L
1
R
500, 100
C
Y
0, 10


15, 250
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