Microeconomic Theory II **Spring 2017**

Midterm Exam **Mikhael Shor**

Solutions

YOUR GRADE IS ON THE LAST PAGE OR BACK OF YOUR EXAM The first three questions were graded out of 20 points, with the rough interpretation that 16+ points is A range and 12+ points is B range. Performance on questions 1 and 3 was generally excellent (vast majority of A's). Question 4 counted as extra credit only. You should especially carefully review any material on questions on which you scored lower than 15 out of 20.
Note: these solutions are not guaranteed to be accurate or complete.

Question 1. Consider the normal form game below.

		Player 2			
		A	B	C	D
Player 1	M	$50, 10$	$58, 20$	$50, 7$	$50, 13$
	N	$40, 5$	$72, 10$	$40, 20$	$32, 12$
	O	$50, 28$	$58, 20$	$60, 10$	$56, 30$
	P	$30, 18$	$64, 20$	$30, 15$	$40, 15$

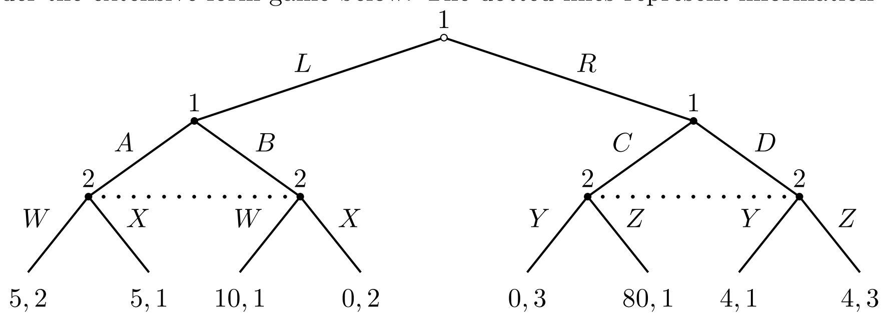
- Is this game dominance solvable? For each step, carefully state and demonstrate which strategies are dominated and which are not.

- P is strictly dominated by various mixtures of the other strategies (e.g., $\frac{1}{2}M+\frac{1}{2}N$).
- Once P is eliminated, A is strictly dominated by D
- Then, M is dominated by mixtures of O and N that put sufficient probability on O (e.g., $\frac{9}{10}O + \frac{1}{10}N$)
- Then B is strictly dominated by D
- Then N is strictly dominated by O
- Then C is strictly dominated by D

Since only O and D remain, the game is dominance solvable.

Two errors students made were assuming that a strategy is dominated as long as it is never a best reply to any *pure* strategy of the opponent and eliminating strategies that are weakly, but not strictly, dominated. Recall that a strategy may be a best response to a mixed (but not pure) strategy of the other player, and a strategy may be dominated by a mixed (but not pure) strategy.

Question 2. Consider the extensive form game below. The dotted lines represent information sets.



(a) Write down or describe the set of all *pure* strategies for each player.

Player 1 has three information sets, so the set of pure strategies is: $\{L, A, C; L, A, D; L, B, C; L, B, D; R, A, C; R, A, D; R, B, C; R, B, D\}$, where each strategy specifies an action at every information set.
Player 2 has two information sets, so the set of pure strategies is: $\{W, Y; W, Z; X, Y; X, Z\}$

(b) Find *all* subgame-perfect Nash equilibria.

The game has two proper subgames, given by:

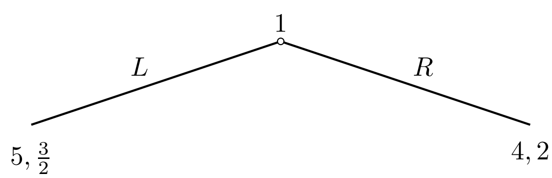
Player 1 \ Player 2	W	X
A	5, 2	5, 1
B	10, 1	0, 2

Player 1 \ Player 2	Y	Z
C	0, 3	80, 1
D	4, 1	4, 3

Each subgame has a unique (mixed strategy) equilibrium, given by: $\{\frac{1}{2}A + \frac{1}{2}B; \frac{1}{2}W + \frac{1}{2}X\}$ and $\{\frac{1}{2}C + \frac{1}{2}D; \frac{19}{20}Y + \frac{1}{20}Z\}$.

Player 1's payoffs in the above subgames are 5 and 4, respectively. This can be seen immediately from the fact that A always earns 5 and D always earns 4.

Next, we analyze the first period.



Player 1 selects L (which earns 5 in the resulting subgame) over R (which earns 4). Therefore, the subgame perfect Nash equilibrium is given by:

$$\{L, \frac{1}{2}A + \frac{1}{2}B, \frac{1}{2}C + \frac{1}{2}D; \frac{1}{2}W + \frac{1}{2}X, \frac{19}{20}Y + \frac{1}{20}Z\}$$

Question 3. Consider a market consisting of a single manufacturer and two retailers. First, the manufacturer sets a wholesale price, w; Second, the two retailers observe w and simultaneously decide on production quantities, q_1 and q_2, which they must purchase from the manufacturer. That is, the two retailers are Cournot competitors with marginal cost w.

The market price is given by $p(q_1, q_2) = 1 - q_1 - q_2$. A retailer's profit is given by $(p - w)q_i, i \in \{1, 2\}$, and the manufacturer's profit is given by $(q_1 + q_2)w$. Each firm is profit maximizing.

- Find the subgame-perfect Nash equilibrium.

First, we find the equilibrium of the second stage for an arbitrary w. Maximizing each retailer's profit with respect to its quantity yields the best response functions:

$$q_1(q_2) = \frac{1}{2}(1 - w - q_2) \quad \textit{and} \quad q_2(q_1) = \frac{1}{2}(1 - w - q_1)$$

Next, we solve the two best response equations simultaneously to find the Nash equilibrium of the second period (which is the Cournot duopoly solution):

$$q_1(w) = q_2(w) = \frac{1}{3}(1 - w)$$

Next, we consider the first period decision for the manufacturer. Since the manufacturer knows what quantities will be chosen for any w, we substitute them out from the manufacturer's profit function:

$$\Pi = (q_1(w) + q_2(w))\, w = \frac{2}{3}(1 - w)w$$

Maximizing the above with respect to w yields $w = \frac{1}{2}$. Therefore, the subgame perfect Nash equilibrium is given by $\{w = \frac{1}{2}; q_1 = q_2 = \frac{1}{3}(1 - w)\}$.

Some student substituted the equilibrium value of w to obtain a specific value for q_1 and q_2. Note that this is the equilibrium outcome, but a subgame perfect Nash equilibrium specifies an action in *every* subgame (i.e., for every w).

Question 4. Imagine that the game below is repeated twice, with the outcome of the first stage observed prior to play in the second. A player's total payoff is the sum of the payoffs of the two periods.

Player 1 \ Player 2	A	B
X	$1,1$	$0,0$
Y	$0,0$	$0,0$

- Does there exist a subgame-perfect Nash equilibrium in which players put strictly positive probability on each strategy in stage 1?

[**Note:** you are not being asked to specify an SPNE or to provide a long solution. A *yes* or a *no* with a few sentences of explanation should suffice.]

To play a mixed strategy, a player must be indifferent between her two strategies. This implies that we need to improve strategy Y and strategy B. The stage game has two Nash equilibria. We can use $\{X, A\}$ as our reward. For example, if $\{Y, B\}$ is played in the second stage following $\{X, A\}$ and $\{X, A\}$ is played following anything that isn't $\{X, A\}$, the first stage looks like this:

Player 1 \ Player 2	A	B
X	$1,1$	$1,1$
Y	$1,1$	$1,1$

Clearly, everything is an equilibrium in the first stage.

Some students seemed to rely on our result that a game *generically* has an odd number of equilibria. Note that this is not a result for every game, but is true with probability 1 for a game created at random (or for this game if each payoff is slightly randomly perturbed, thus eliminating ties and weak dominance).