Microeconomics III **Midterm Exam Spring 2012**

Solutions

YOUR GRADE IS ON THE LAST PAGE OF YOUR BLUEBOOK

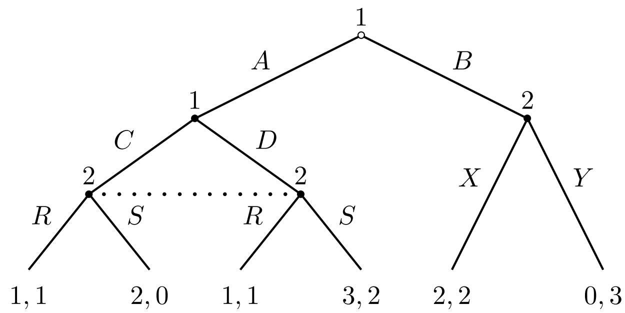
Question 1. Consider the following game.

		Player 2			
		A	B	C	D
Player 1	M	3, 2	2, 0	2, 3	2, 6
	N	2, 2	1, 4	3, 3	4, 2
	O	3, 3	4, 1	2, 4	1, 3
	P	1, 5	0, 4	2, 3	3, 5

(a) What strategies survive the iterated deletion of strictly dominated strategies? For each iteration, specify the dominated strategy and the strategy that dominates it.
(i) P is dominated by N, (ii) A is dominated by C
(iii) M is dominated by $\frac{1}{2}N, \frac{1}{2}O$, (iv) D is dominated by C
Two notes:
(i) recall that a strategy cannot be dominated if it is ever a best response. Thus, you can restrict attention only to strategies that are never a best response to any pure strategy, and see if they are dominated, (ii) recall that a strategy might not be dominated by a pure strategy but is nevertheless dominated by a mixed strategy (as in (iii) once P and A are eliminated).

(b) What is the unique Nash equilibrium of this game?
$\{ \frac{3}{4}N, \frac{1}{4}O; \frac{1}{4}B, \frac{3}{4}C \}$
The game does not have a pure strategy equilibrium, thus we know that the equilibrium has to be mixed. After eliminating dominated strategies (which cannot be played with positive probability), calculate each player's probability of each strategy that makes the *other* player indifferent between his strategies. If you only narrowed the game down to three strategies for each player, you could still obtain the right answer, though with more work. In particular, you would find that a mixed strategy in which all three strategies are used with positive probability cannot exist.

Question 2. Consider the game below. Both the extensive form and the normal form are given. The dotted line represents an information set.



		Player 2			
		R,X	R,Y	S,X	S,Y
Player 1	A,C	1, 1	1, 1	2, 0	2, 0
	A,D	1, 1	1, 1	3, 2	3, 2
	B,C	2, 2	0, 3	2, 2	0, 3
	B,D	2, 2	0, 3	2, 2	0, 3

(a) List all pure-strategy Nash equilibria.
There are three: {A,C;R,Y}, {A,D;S,X}, {A,D;S,Y}

(b) List all pure-strategy trembling-hand perfect equilibria.
{A,D;S,Y} is the only THPE
Recall that, for a two player finite game, THPE is equivalent to any Nash equilibrium in which no player is playing a weakly dominated strategy. Generally, a weakly dominated strategy can never be part of a THPE (and the equivalence holds for two-player games). Thus, to determine the THPE, one only needs to check for each of the three Nash equilibria if either strategy is weakly dominated.
{A,C;R,Y}: A,C is weakly dominated by B,C
{A,D;S,X}: S,X is weakly dominated by S,Y
{A,D;S,Y}: neither A,D nor S,Y is weakly dominated

(c) List all pure-strategy subgame-perfect equilibria.
{A,C;R,Y} and {A,D;S,Y}
An SPNE is a Nash equilibrium that induces a Nash equilibrium in every subgame. There are two proper subgames: (i) the left one, with pure-strategy equilibria {C,R} and {D,S}, and (ii) the right one, with pure strategy equilibrium {Y}. Therefore, checking our pure-strategy Nash equilibria:
{A,C;R,Y}: Both {C,R} and {Y} are Nash equilibria of their subgames,
{A,D;S,Y}: Both {D,S} and {Y} are Nash equilibria of their subgames,
{A,D;S,X}: {X} is not a Nash equilibrium of its subgames,
Of course, one can also solve the problem through backward induction by first checking for equilibria in each subgame, and then seeing what player 1 does at the first node. However, remember that multiple equilibria in a subgame implies that we have multiple subgame perfect equilibria, corresponding to each possible equilibrium in the subgame.

(d) List all pure-strategy sequential equilibria.
There are two: (i) {A,C;R,Y}, $\mu(C) = 1$ and (ii) {A,D;S,Y}, $\mu(D) = 1$
Recall that a sequential equilibrium consists of both strategies *and* beliefs, and a sequential equilibrium induces a subgame perfect equilibrium. Therefore, we only need to check our two subgame perfect equilibria, prescribe consistent beliefs, and see if behavior is a best reply to those beliefs. In this case, both pure-strategy subgame perfect equilibria prescribe action A at the first node, and thus player 2's information set is always on the equilibrium path. Player 1's strategy A,C implies that player 2 must believe the probability of being at the left node of his information set is 1.

Question 3. Three students ($i = 1, 2, 3$) are working on a joint project. Each student selects the amount of time to work on the project, $x_i \geq 0$. The quality of the project, q, depends on x_1, x_2, and x_3:

$$q(x_1, x_2, x_3) = 3x_1 + 2x_2 + 2x_3 - x_1x_2 - x_1x_3$$

Each student also has a cost function given by

$$c_i(x_i) = (x_i)^2$$

The utility for student i is given by:

$$u_i(x_1, x_2, x_3) = q(x_1, x_2, x_3) - c_i(x_i)$$

(a) Suppose that the students simultaneously and independently decide how much time to spend on the project. Show each student's best response function and determine the pure-strategy Nash equilibrium of this game.
To determine the best response functions, we maximize each player's utility with respect to that player's strategy. We obtain:
$BR_1(S_{-1}) = x_1(x_2, x_3) = \frac{1}{2}(3 - x_2 - x_3)$
$BR_2(S_{-2}) = x_2(x_1, x_3) = \frac{1}{2}(2 - x_1)$
$BR_2(S_{-2}) = x_3(x_1, x_2) = \frac{1}{2}(2 - x_1)$
The Nash equilibrium is the solution to the above three equations given by $x_1 = 1, x_2 = x_3 = \frac{1}{2}$.

(b) Consider the following two-period game. First, student 1 decides how much time to spend on the project. Second, after observing the choice of student 1, students 2 and 3 simultaneously and independently decide how much time to spend. Determine the pure-strategy subgame-perfect Nash equilibrium of this game.
The SPNE is: $x_1 = 0, x_2 = x_3 = 1 - \frac{1}{2}x_1$ (which, in equilibrium, implies $x_2 = x_3 = 1$).
Player 1 first chooses x_1, then players 2 and 3 simultaneously choose x_2 and x_3. Thus, this game has a proper subgame for *every* possible choice of x_1.
In the second period, given first period choice of x_1, players 2 and 3 play $x_2 = x_3 = \frac{1}{2}(2-x_1)$ (see above).
In the first period, player 1 maximizes his utility *given* the response of players 2 and 3.
Thus, player 1 maximizes:
$u_1(x_1, x_2(x_1), x_3(x_1))$, where $\frac{1}{2}(2 - x_1)$ are substituted for x_2 and x_3 in his utility function. This yields $u_1(x_1, x_2(x_1), x_3(x_1)) = 4 - x_1$, which is maximized at $x_1 = 0$ (given the constraint $x_1 \geq 0$).

Question 4 (If time, save for last). Two firms are deciding on which technology to adopt, A or B. Technology A is only profitable if adopted by both firms. The payoffs based on their decisions are given by The game is played over two periods. Each firm may make an adoption decision only once, but may do so in

Firm 1 \ Firm 2	A	B
A	$3, 3$	$0, 2$
B	$4, 0$	$1, 1$

either period 1 or period 2. Players move simultaneously in each period (though may select no action, N, in period 1), and first period choices are observed before a second-period action is chosen (if N were chosen in period 1). Payoffs are received after the second period.

(a) Is there a subgame-perfect equilibrium in which (A,A) is ultimately chosen? If yes, what is it? If no, why not?

(b) Find all subgame-perfect equilibria of this game. [Hint: there are nine possible combinations of first-period actions. What are the payoffs from each of these when second-period actions are taken into account?]
Each player can select A,B, or N in period 1. Then, players who selected N in period 1 must select either A or B in period 2 (players who selected A or B in period 1 have no choice to make in period 2). There are nine possible pairs of actions taken in round 1:
{A,A}, {A,B}, {B,A}, {B,B}: these have no subgames following, as there are no choices to make in period 2
{N,A}, {N,B}: these have a choice for player 1 in period 2, in which player 1 chooses B (his best response to either A or B).
{A,N}: this leaves a choice for player 2 in period 2, in which player 2 chooses A (his best response to A).
{B,N}: this leaves a choice for player 2 in period 2, in which player 2 chooses B (his best response to B).
{N,N}: this has a subgame following equivalent to the game drawn above, which has unique equilibrium {B,B}. Compiling this information together, and (in the spirit of subgame perfection), replacing second period subgames with their equilibrium outcomes, the first-period game looks like this:

Firm 1 \ Firm 2	A	B	N
A	$3, 3$	$0, 2$	$3, 3$
B	$4, 0$	$1, 1$	$1, 1$
N	$4, 0$	$1, 1$	$1, 1$

For example, if player 1 chooses N and player 2 chooses A in the first period, player 1 in the second period will choose B. Thus, ultimately, they will end up with B for player 1 and A for player 2, which has payoffs $4, 0$.
The first-stage game above has three pure-strategy equilibria: {B,B}, {N,B}, and {A,N}. The subgame perfect equilibria corresponding to each consist of the first-stage moves and the description above of the second-stage actions. For part a of the question, note that if player 1 selects A and player 2 selects N in the first period, player 2 will then select A in the second period.