Microeconomics III **Spring 2013**

Midterm Exam **Mikhael Shor**

Solutions

YOUR GRADE IS ON THE LAST PAGE OF YOUR BLUEBOOK Each question was graded out of 10 points. Due to time constraints, the lowest grade was dropped and the highest grade counted in its place (twice). Question 1 had the best overall performance, followed by question 3. You should especially carefully review any material on questions on which you scored lower than 7 out of 10. Roughly speaking, 34+ is probably in the A range, 25+ in the B range.
Note: these solutions are not guaranteed to be accurate or complete.

Question 1. Consider the normal form game below.

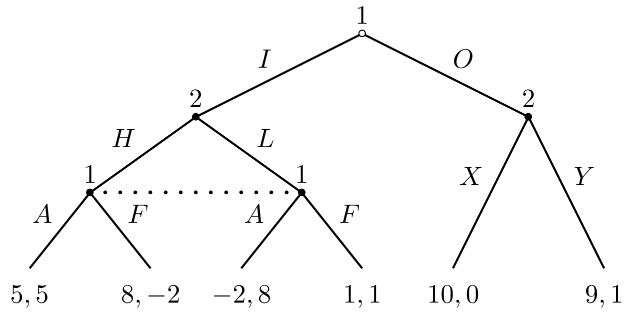
Player 2

Player 1		A	B	C	D
	M	8, 3	7, 2	7, 4	7, 7
	N	7, 3	6, 6	4, 4	9, 3
	O	8, 4	8, 2	8, 8	7, 4
	P	6, 6	4, 4	3, 4	8, 6

(a) What strategies are weakly dominated?
`M (by O), P (by N), A (by D)`
`Notes:`
• `P is strictly dominated by N, which implies that it is weakly dominated.`
• `Determining dominated strategies does not require the iterated deletion of strictly dominated strategies. If a strategy is not dominated unless other strategies are first deleted, then it is not dominated.`

(b) What strategies survive the iterated deletion of strictly dominated strategies? Carefully explain each step.
`Only O and C survive, which implies that the game is dominance solvable (a unique solution is obtained).`
`Steps: P (by N), A (by C), M by qO+(1-q)N for` $1 > q > \frac{3}{4}$`, D (by C), N (by O), B (by C)`
`Notes:`
• `In the third step, M is not dominated by any pure strategy, but it is dominated by an appropriate mixed strategy.`
• `We do not delete weakly dominated strategies in this iterative process.`
`Recall that equilibria may involve the playing of weakly dominated strategies.`
• `One could eliminate strategies that are never a best response by invoking the theorem that rationalizable and iterated deletion are identical for two player games. However, confirming something is never a best response requires showing that it is not a best response to any strategies of the other players, including mixed.`

Question 2. Consider the game below. Both the extensive form and the normal form are given. The dotted line represents an information set.



		Player 2			
		H, X	H, Y	L, X	L, Y
Player 1	I, A	5, 5	5, 5	−2, 8	−2, 8
	I, F	8, −2	8, −2	1, 1	1, 1
	O, A	10, 0	9, 1	10, 0	9, 1
	O, F	10, 0	9, 1	10, 0	9, 1

(a) What strategies are rationalizable? `I,A and I,F are both strictly dominated, and therefore both are never a best reply. Once eliminated, H,X and L,X are strictly dominated, and therefore both are never a best reply.`
`This leaves O,A ; O,F ; H,Y ; and L,Y.`

(b) List all pure-strategy Nash equilibria. {`O,A ; H,Y`} , {`O,F ; H,Y`} , {`O,A ; L,Y`} , {`O,F ; L,Y`}

(c) List all pure-strategy subgame-perfect equilibria. {`O,F ; L,Y`}.
`The unique equilibrium of the subgame on the left, pictured below (a subset of the normal form given above) is` {`F,L`} .

		Player 2	
		H	L
Player 1	A	5, 5	−2, 8
	F	8, −2	1, 1

`The unique equilibrium of the subgame on the right is Y. Of the four Nash Equilibria listed above, only one induces a N.E. in each subgame.`

Question 3. Consider the game above.

		Player 2	
		A	B
Player 1	X	$1+a,-1$	$-1,1$
	Y	$-1,1$	$1,-1$

(a) Find the Nash equilibrium for $a \geq 0$.
For player 1 to be indifferent between X and Y, player 2 must assign probability $\frac{2}{4+a}$ to strategy A. For Player 2 to be indifferent between A and B, player 1 must assign equal probability to X and Y. Therefore, the equilibrium is
$\{\frac{1}{2}X + \frac{1}{2}Y, \frac{2}{4+a}A + \frac{2+a}{4+a}B\}$

(b) Consider a two-stage game where (i) player 1 chooses a at a cost of $a/16$ and then (ii) both players observe a and play the above simultnaeous-move game. Find the unique subgame perfect equilibrium.

Each possible choice of a defines a subgame. We have already determined that the game has a unique Nash equilibrium in each of these subgames, given above. The payoff in a subgame for player 1 is given by
$\frac{2}{4+a}(1+a) + \frac{2+a}{4+a}(-1) = \frac{a}{4+a}$
Therefore, in the first period, player 1 maxmizes
$\frac{a}{4+a} - \frac{a}{16}$ which gives $a = 4$.
For completeness, recal that a subgame perfect Nash equilibrium specifies an action in every subgame. Thus, the SPNE is: $\{a = 4, \frac{1}{2}X + \frac{1}{2}Y; \frac{2}{4+a}A + \frac{2+a}{4+a}B\}$
Notes:
• Any reasonable answer to this question must involve player 1 selecting a value of a, maximizing profit given the equilibrium of the second stage.
• Some people subtracted $a/16$ from each of player 1's payoffs in the normal form game instead. This is okay, but (i) this will obviously not change the mixed strategy equilibrium (adding a constant to each payoff), and (ii) you still have to determine the first-stage strategy, or value of a, that maximizes player 1's payoff.
• Some people subtracted $a/16$ from player 1's payoffs only in the outcome {X,A}. This does not make much sense -- player 1 selects an a and pays for it only when random play ensures he gets it? This violates the timing of the game, where player 1 selects a at a cost of $a/16$ and THEN the game is played. Either way, under this assumption player 1 pays $\frac{a}{16}$ only when he receives $1+a$; therefore, player 1 will select as large of a value of a as possible and earn infinite payoffs.

(c) What is each player's expected payoff in the subgame perfect equilibrium? Does player 2's payoff change with a? Briefly explain intuitively why or why not.

From above, player 1's payoffs are given by $\frac{a}{4+a} - \frac{a}{16}$ evaluated at $a = 4$ which yields $\frac{1}{4}$. Player 2's expected payoff is zero (in every subgame). Mixed strategies assure that each player is indifferent between her strategies. As player 2's payoffs do not vary with a, the indifference condition does not, either.
Note that simply saying ''a does not enter player 2's payoffs'' is insufficient. This could nevertheless change the equilibrium (and thus player 2's payoffs). It is the existence of a mixed strategy equilibrium that makes this case. For example, consider $a = -10$. The problem did not allow this, but it illustrates that player 2's payoffs can change with a, as player 2 can now earn 1 in equilibrium.

Question 4. An industry consists of two symmetric firms, A and B, who are Cournot duopolists. Inverse demand is given by $P = 60 - q_A - q_B$, where q_A and q_B are the output decisions of firms A and B, respectively. Each firm has a constant marginal cost of 12. The revenue of firm $i \in \{A, B\}$ is given by Pq_i and the profit by $(P - 12)q_i$.

Firm B is managed by its owner who selects q_B to maximize profit of Firm B.

Firm A has both an owner and a manager. The manager selects q_A. However, prior to the quantity competition stage, the owner of Firm A determines how to compensate the manager. The manager of Firm A selects q_A to maximize his income (given the contract chosen by his owner) and the owner of Firm A maximizes the firm's net income (profit minus manager compensation).

The game proceeds in two stages. In the first stage, the owner of Firm A announces either a profit-sharing or revenue-sharing contract for its manager. A profit-sharing contract gives the manager of firm A a fraction β of the profit of the firm while a revenue-sharing contract gives the manager a fraction β of the revenue of the firm. The fraction β, $0 < \beta < 1$, is fixed and exogenously given. Then, in the second stage, the managers simultaneously select q_A and q_B.

1. Find the Nash equilibrium of the second stage of this game under a profit-sharing contract for any β.

   Profits: $\pi_A = \beta(60 - 12 - q_A - q_B)q_A$, $\pi_B = (60 - 12 - q_A - q_B)q_B$
   Best responses: $q_i(q_j) = 24 - \frac{1}{2}q_j$
   Equilibrium: $q_A = q_B = 16$.

2. Find the Nash equilibrium of the second stage of this game under a revenue-sharing contract for any β.

   Profits: $\pi_A = \beta(60 - q_A - q_B)q_A$, $\pi_B = (60 - 12 - q_A - q_B)q_B$
   Best responses: $q_A(q_B) = 30 - \frac{1}{2}q_B$, $q_B(q_B) = 24 - \frac{1}{2}q_A$
   Equilibrium: $q_A = 24, q_B = 12$.

3. Suppose that β is arbitrarily small. What is the subgame perfect Nash equilibrium of this game?

   Many students simply noted that the equilibria above are independent of β. This is not relevant. The owner in the first stage is not selecting β, but is selecting which of the above two second-period outcomes he prefers. The two outcomes do differ, and therefore we must compare the owner's profit under each.

   Under profit sharing, the owner's profit is given by $(1 - \beta)(60 - 12 - q_A - q_B)q_A = (1 - \beta)(60 - 12 - 16 - 16)16 = 256(1 - \beta)$

   Under revenue sharing, the owner's profit is given by $(60 - 12 - q_A - q_B)q_A - \beta(60 - q_A - q_B)q_A = (60 - 12 - 24 - 12)24 - \beta(60 - 24 - 12)24 = (12)24 - (24)24\beta = 288(1 - 2\beta)$

   Thus, for sufficiently small β, the owner prefers the revenue-sharing contract to the profit-sharing contract.

   Easier math would just compare total profit of firm A under profit sharing $(p-12)q_A = 256$ and revenue sharing $(p - 12)q_A = 288$. With β sufficiently small, the profit goes almost entirely to the owner.

   Thus, the SPNE is { Revenue Sharing; $q_A = 24, q_B = 12$ (rev sharing); $q_A = q_B = 16$ (prof sharing) } where the last part accounts for the NE in the subgame where prof sharing is chosen.

4. How does delegation impact the owner's profit? Briefly explain and provide some intuition.

   Note that the form of the delegation changes the manager's incentives. When maximizing revenue, instead of profit, the manager does not account for the cost of each unit and thus overproduces. However, the rival firm anticipates the overproduction in equilibrium and thus reduces its output. In the case of profit sharing, we have the Cournot outcome, with each firm producing 1/3 of the competitive outcome. In the case of revenue-sharing, we have an outcome similar to Stackelberg, where the revenue-sharing firm produces a majority of the output. Therefore, the owner of the firm prefers a contract that gives the manager incentive to overproduce.