Microeconomic Theory II   Spring 2014

YOUR GRADE IS ON THE LAST PAGE OF YOUR BLUEBOOK
MIDTERM SOLUTIONS: Done quickly, only illustrative; they are neither complete nor guaranteed. See grading note at end of solutions.

Question 1. Consider the following game.

		Player 2			
		A	B	C	D
Player 1	W	30, 30	50, 10	20, 40	10, 30
	X	10, 50	0, 40	10, 30	30, 50
	Y	30, 20	20, 10	10, 30	10, 60
	Z	20, 20	10, 40	30, 30	50, 20

(a) What strategies are weakly dominated? X and Y for player 1, and A for player 2. X is strictly (and therefore, weakly) dominated by Z, Y is weakly dominated by W, and A is weakly dominated by D.

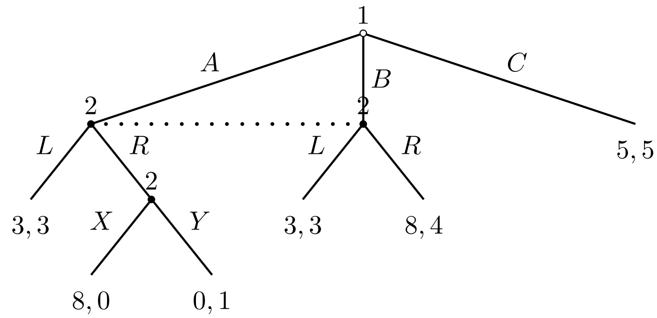
(b) What strategies survive the iterated deletion of strictly dominated strategies? For each iteration, specify the dominated strategy and the strategy that dominates it. Strategies {W,Z,B,C} survive. First, X is strictly dominated by Z. Second, A is strictly dominated by C. Third, Y is strictly dominated by $\frac{1}{2}$W $+\frac{1}{2}$Z. Fourth, D is strictly dominated by C.
Notes: We do not (iteratively) eliminate strategies that are weakly dominated. Also, in a two-player game, a strategy is strictly dominated if and only if it is never a best response; however, demonstrating that a strategy is never a best response to any opponent strategy, not only any pure strategy.

(c) Is the game dominance solvable? Explain. It is not. Dominance solvable means that the process of iterated deletion of strictly dominated strategies results in a single *outcome* (a single pure strategy for each player).

(d) What is the unique equilibrium of this game? $\{\frac{1}{4}W + \frac{3}{4}Z; \frac{1}{5}B + \frac{4}{5}C\}$. By inspection, the game does not have a pure strategy equilibrium. Further, after eliminating the above games, we have only two pure strategies remaining for each player. Making each player indifferent between her two strategies yields the equilibrium.

(e) Imagine that the above game is repeated twice. Write down one (any) pure strategy subgame perfect equilibrium of this repeated game. NOTE that this was an error on my part as the pure strategy requirement was for a previous version of this question that I decided not to use. There is no pure strategy SPNE. Students were not unduly penalized due to my error. The solution:the only SPNE in any finite repetition of a stage game with a unique Nash equilibrium is the play of that NE in every subgame.

Question 2. Consider the game below. Both the extensive form and the normal form are given. The dotted line represents an information set.



		Player 2			
		L, X	L, Y	R, X	R, Y
	A	3, 3	3, 3	8, 0	0, 1
Player 1	B	3, 3	3, 3	8, 4	8, 4
	C	5, 5	5, 5	5, 5	5, 5

(a) List all pure-strategy Nash equilibria.
{C;L,X}, {C;L,Y}, {B;R,X}, {B;R,Y}

(b) List all pure-strategy subgame-perfect Nash equilibria.
The *only* proper subgame in this game is player 2's decision between X and Y, in which he chooses Y. As a SPNE prescribes a NE in each subgame, this is simply the set of Nash equilibria that prescribe Y. Therefore, {C;L,Y} and {B;R,Y} are the only pure-strategy SPNE.
Note that there is not a subgame containing only player 1's actions A and B. A subgame begins with a singleton node and includes *every* successor. Also, there is no subgame starting at player 2's information set.

(c) Briefly discuss which of the above Nash equilibria are also trembling-hand perfect equilibria.
Trembling hand equilibria rule out weakly dominated strategies and thus rule out A and R,X. This means that {B;R,X} is a NE but is not trembling hand perfect, wile the other NE all are.

(d) Briefly explain or demonstrate whether this game has any mixed strategy subgame-perfect Nash equilibria.
There are many ways to approach this problem, but first eliminate the strategies L,X and R,X (as they cannot be subgame perfect) and then A (as it is now dominated by C). For a simple mixed strategy equilibrium, note that when player 1 plays C, player 2 is indifferent and thus can put some probability on R,Y that is sufficiently small to maintain C as a best response for player 1.

Question 3. Two firms produce an identical good. Inverse demand is given by P=32-Q. Competition between the two firms occurs in two stages:

- In stage 1, firms simultaneously choose whether to use the old or the new technology to produce their goods. The old technology has a marginal cost of 20 but requires no investment. The new technology reduces marginal cost to 8 but requires a fixed investment of 50.
- After the decisions from stage 1 are made public, in stage 2, both firms simultaneously choose a quantity.

Effectively, stage 2 is a two-firm Cournot game in which each firm has the marginal costs it chose in stage 1.

(a) Find the Nash equilibrium and resulting profits of the stage 2 subgame after both firms have chosen the *old* technology.

(b) Find the Nash equilibrium and resulting profits of the stage 2 subgame after both firms have chosen the *new* technology.

(c) Find the Nash equilibrium and resulting profits of the stage 2 subgames after the firms have chosen different technologies.

We can solve the above three for generic costs. Let firm 1 have costs of c_1 and firm 2 have costs of c_2. Then:

$$\Pi_i(q_i, q_j) = (32 - q_i - q_j - c_i)q_i$$

Differentiating with respect to q_i and setting equal to zero yields the following best response functions:

$$q_1 = 16 - \frac{1}{2}q_2 - \frac{1}{2}c_1 \text{ and } q_2 = 16 - \frac{1}{2}q_1 - \frac{1}{2}c_2$$

which yields the equilibrium

$$q_1 = \frac{32 - 2c_1 + c_2}{3} \text{ and } q_2 = \frac{32 + c_1 - 2c_2}{3}$$

Substituting, we obtain for (a): $q_1 = q_2 = 4$, (b): $q_1 = q_2 = 8$, and, assuming firm 1 uses the old technology for (c): $q_1 = 0, q_2 = 12$.
Note that for part (c), one firm has the same best response function as in (a) and the other has the same best response function as in (b), but these do not imply that the equilibrium quantities are the same as in (a) and (b)!

(d) Find all subgame perfect Nash equilibria of this game. Calculating the equilibrium profits from each of the above scenarios yields:

		Player 2	
		new	*old*
Player 1	*new*	14, 14	94, 0
	old	0, 94	16, 16

The unique equilibrium of the above first-stage game is {new,new} and therefore the subgame perfect equilibrium consists of each player selecting new in the first stage and, for *each* of the four resulting subgames, actions given by the four subquestions above.

(e) Briefly discuss, intuitively, what type of game this appears to be, whether the outcome is efficient, and why or why not. This is akin to a prisoner's dilemma (usually, a PD has off-diagonal elements that sum to less than the diagonal elements). Each has a dominant strategy to adopt the new technology, but the resulting profits are lower than if both did not.

(f) (A <u>brief</u> and intuitive answer is likely sufficient for this:) Imagine that stage 2 involved Bertrand competition (in prices) instead of Cournot competition. What would the subgame perfect equilibria look like? Note that in Bertrand competition, both firms earn zero profits when they have the same costs. However, when firms have different costs, one can undercut the other (i.e., the firm with marginal costs of 8 can charge just under the other firm's marginal cost of 20 resulting in 12 sales and profit of $144 - 50 = 94$). Therefore, the resulting stage 1 game is:

		Player 2	
		new	*old*
Player 1	*new*	$-50, -50$	$94, 0$
	old	$0, 94$	$0, 0$

Each has a best response to adopt the opposite technology from its rival.

GRADING Each question was graded out of 40. My handwriting is horrible, which is why I supply these solutions. Obviously, if you can't make out something I wrote on your exam, come see me. Grades were quite trimodal. An A grade would require a score in the 100s (out of 120). A B would likely require something in the mid 80s, indicating that some review woulf be valuable. Grades substantially below 70 are quite worrisome.