Microeconomic Theory II — Spring 2015

Midterm Exam — Mikhael Shor

Read all questions carefully and work carefully.
Be explicit about any assumptions that you are making.

Question 1. Consider the following normal-form game:

		Player 2		
		R	S	T
	A	$5, 5$	$8, 3$	$1, 1$
Player 1	B	$3, 2$	$6, 6$	$2, 5$
	C	$2, 0$	$3, 0$	X, Y

(a) For what values of X and Y is the game above dominance solvable?

For the remainder of this question, imagine that the stage game above is repeated twice, with the outcome of the first period observed by both players before the second period, and with payoffs equal to the sum of the two periods (i.e., no discounting).

(b) For what values of X and Y does the game have a pure-strategy subgame-perfect Nash equilibrium in which players play {A,R} in the first period? Explain.

(c) For what values of X and Y does the game have a pure-strategy subgame-perfect Nash equilibrium in which players play {B,S} in the first period? Explain.

(d) For what values of X and Y does the game have a pure-strategy subgame-perfect Nash equilibrium in which players play {C,T} in the first period? Explain.

Question 2. A diamond seller can sell either a real or a fake diamond. A buyer can pay for an independent inspection of the diamond or simply buy it without inspecting it. If a seller is caught (through inspection) selling a fraudulent diamond, he must pay a fine of $F > 0$ to the government. The payoffs are:

		Buyer	
		Inspect	Don't Inspect
Seller	Real	$2, 2$	$3, 3$
	Fake	$3 - F, 1$	$4, 0$

(a) What are the Nash equilibria of the above game (this will depend on F)?

(b) What are the seller's and the buyer's expected equilibrium payoffs?

(c) How do the buyer's and seller's equilibrium payoffs change with F?

(d) The government's expected revenue is equal to the fine, F, times the equilibrium probability that the fine is collected (when the buyer inspects a fake diamond). Describe or sketch the government's expected revenue as a function of F.

Question 3. Consider n identical fishermen who fish in the same lake. Every year, each fisherman receives an income of 1 and must choose how much of this income to contribute to stocking the lake with fish for the upcoming season. There is no saving and contribution decisions are made simultaneously in each year. Stocking the lake helps all fishermen; for every dollar invested in stocking fish, *each* fisherman receives α dollars in additional revenue, $\frac{1}{n} < \alpha < 1$ (e.g., $\alpha = 0.7$).

Denote by m_i, $0 \leq m_i \leq 1$, fisherman i's contribution. Thus, the utility of fisherman i is:

$$u_i = 1 - m_i + \alpha \sum_{j=1}^{n} m_j$$

(a) Find the Nash equilibrium of this game.

(b) Find the socially-optimal level of investment for each fisherman (i.e., the level of investment by each that maximizes total welfare).

(c) Suppose that this game is infinitely repeated, with future payoffs discounted by δ.
Each fisherman adopts the following strategy:

- Contribute 1 in the first year and in every subsequent year if, in all previous years, everyone contributed 1.
- Contribute 0 If, in any previous year, anyone contributed less than 1.

For what range of δ are these strategies a subgame-perfect Nash equilibrium of the infinitely-repeated game?

(d) How does your answer in (c) change with n? Does this make sense? Why or why not?