Microeconomic Theory II **Spring 2019**
Midterm Exam **Mikhael Shor**

Solutions

YOUR GRADE IS ON OR NEAR THE LAST PAGE OF YOUR EXAM
Each question was graded out of 20 points, with the rough interpretation that 17+ points is A range and 12+ points is B range. Performance on all three questions was consistent (average of around 15). You should especially carefully review any material on questions on which you scored lower than 15 out of 20.
Note: these solutions are not guaranteed to be accurate or complete.

Question 1. Consider the normal form game below.

		Player 2			
		A	B	C	D
Player 1	M	$10,1$	$10,\ 0$	$10,\ 0$	$80,\ 0$
	N	$100,1$	$20,\ 6$	$10,\ 18$	$40,\ 0$
	O	$60,1$	$10,\ 24$	$30,\ 0$	$20,\ 0$
	P	$88,4$	$16,500$	$14,500$	$18,500$

(a) What strategies survive the iterated deletion of strictly dominated strategies? Be sure to demonstrate why a strategy is dominated.

Solution:

- P is dominated by $\frac{3}{4}N + \frac{1}{4}O$, along with a range of other mixed strategies involving N and O.
- D is dominated by A
- M is dominated by any mixture of N and O with strictly positive probability on each.
- A is strictly dominated by B
- None of the remaining strategies are strictly dominated, so B, C, N, and O survive IDSDS.

(b) Find all Nash equilibria.

Solution: There is no pure-strategy equilibrium. The unique equilibrium is given by:
$\{\frac{2}{3}N + \frac{1}{3}O; \frac{2}{3}B + \frac{1}{3}C\}$

(c) Imagine that this game is repeated twice. Is there a subgame-perfect equilibrium in which Player 2 earns 500 in the first period? Carefully explain why or why not.

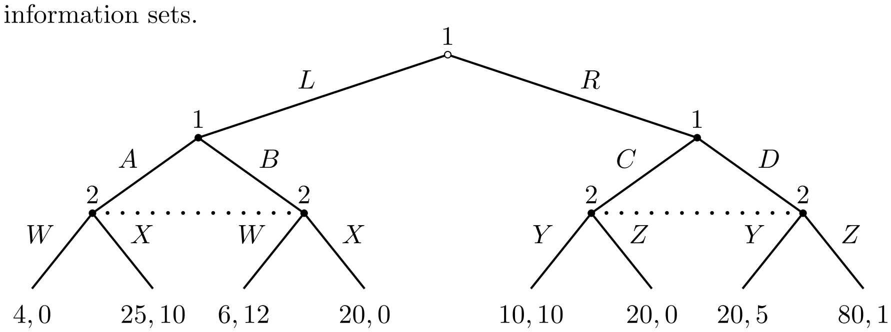
(d) Imagine that this game is repeated an infinite number of times. Is there a subgame-perfect equilibrium in which Player 2 earns an average payoff of 500? Carefully explain why or why not.

Solution: If a stage game has a unique NE, then the subgame-perfect NE of any finite repetition of that stage game involves playing the NE in every stage. Intuitively, we need multiple equilibria to alternately "reward" or "punish" players for taking certain actions in the first round. Since this stage game has a unique NE (which does not earn Player 1 500) it is not possible to earn 500 in a twice-repeated version.

For infinitely repeated games, it may be possible to achieve alternate outcomes even when there is a unique equilibrium and even if players have dominated strategies. For example, we saw how the infinitely repeated prisoners dilemma gives rise to multiple subgame-perfect equilibria. In fact, the folk theorem tells us that many such results are possible when players are sufficiently patient. The key is checking which outcomes are and which are not sustainable, which depends on the specific payoffs. This could be done either by constructing such an equilibrium with the appropriate punishment strategy (the stage game equilibrium payoff) or by appeal to the folk theorem. I gave full credit to any answer that contemplated (regardless of the conclusion) the tradeoffs between cooperating, on one hand, and capturing higher one-time gains by not cooperating followed by equilibrium payoffs. For example, to coordinate on $\{P, D\}$, we must avoid player 1 deviating to M. We could threaten to revert to the equilibrium payoff forever (like the grim trigger strategy). This would induce cooperation if:
$\frac{18}{1-\delta} > 80 + \frac{50}{3}\frac{\delta}{1-\delta}$

Note that our answer for (c) depends critically on this stage game having a unique NE. Otherwise, it may be possible depending on the structure of the game. Thus, generic answers (e.g., about how P is dominated and therefore cannot be played in the first round or how no stage-game equilibrium involves P) that suggest that it wouldn't be possible in general or answers that ignore the difference between repeated and one-shot games are not correct.

Question 2. Consider the extensive form game below. The dotted lines represent information sets.



(a) Write down or describe the set of all *pure* strategies for each player.

Solution: Player 1 has *three* information sets, and thus a strategy for player 1 consists of an action at *each* of them. The set of pure strategies is given by:

$\{L, R\} \times \{A, B\} \times \{C, D\} \equiv \{LAC, LAD, LBC, LBD, RAC, RAD, RBC, RBD\}$

And for player 2: $\{W, X\} \times \{Y, Z\} \equiv \{WY, WZ, XY, XZ\}$

(b) Find all pure-strategy subgame-perfect Nash equilibria.

Solution: Note that nowhere in this question are you asked for Nash equilibria, but only for subgame-perfect Nash equilibria. Therefore, we can use backward induction rather than try to find the Nash equilibria of the whole ($8x4$) game. The game has two proper subgames.

- The left subgame has two pure-strategy Nash equilibria: $\{A, X\}$ and $\{B, W\}$
- The right subgame has one pure-strategy Nash equilibrium: $\{D, Y\}$

This implies that the overall game will have (at least) two pure-strategy subgame-perfect Nash equilibria: one corresponding to each of the NE in the left subgame:

- In the case of $\{A, X\}$ and $\{D, Y\}$, player 1 would choose L, so one equilibrium is $\{L, A, D; X, Y\}$
- In the case of $\{B, W\}$ and $\{D, Y\}$, player 1 would choose R, so one equilibrium is $\{R, B, D; W, Y\}$

Question 3. Consider a market consisting of a single manufacturer and two retailers. First, the manufacturer sets a wholesale price, w. Second, the two retailers observe w and simultaneously decide on production quantities, q_1 and q_2, which they must purchase from the manufacturer. That is, the two retailers are Cournot competitors with marginal cost w.
The market price is given by $p(q_1, q_2) = 1 - q_1 - q_2$. A retailer's profit is given by $(p - w)q_i, i \in \{1, 2\}$, and the manufacturer's profit is given by $(q_1 + q_2)w$. Each firm is profit maximizing.

- Find the subgame-perfect Nash equilibrium.

Solution: First, we find the equilibrium of the second stage for an arbitrary w. Maximizing each retailer's profit with respect to its quantity yields the best response functions:

$$q_1(q_2) = \frac{1}{2}(1 - w - q_2) \quad \textit{and} \quad q_2(q_1) = \frac{1}{2}(1 - w - q_1)$$

Next, we solve the two best response equations simultaneously to find the Nash equilibrium of the second period (which is the Cournot duopoly solution):

$$q_1(w) = q_2(w) = \frac{1}{3}(1 - w)$$

Next, we consider the first period decision for the manufacturer. Since the manufacturer knows what quantities will be chosen for any w, we substitute them out from the manufacturer's profit function:

$$\Pi = (q_1(w) + q_2(w))\, w = \frac{2}{3}(1 - w)w$$

Maximizing the above with respect to w yields $w = \frac{1}{2}$. Therefore, the subgame perfect Nash equilibrium is given by $\{w = \frac{1}{2}; q_1 = q_2 = \frac{1}{3}(1 - w)\}$.

Some student substituted the equilibrium value of w to obtain a specific value for q_1 and q_2. Note that this is the equilibrium outcome, but a subgame perfect Nash equilibrium specifies an action in *every* subgame (i.e., for every w).