Microeconomic Theory II **Spring 2020**

Midterm Exam **Mikhael Shor**

Solutions

Question 1. Consider the following game.

Player 2

Player 1		A	B	C	D
	M	8, 0	2, 4	500, 1	20, 1
	N	4, 4	16, 0	8, 1	10, 0
	O	8, 3	1, 100	12, 50	30, 90
	P	2, 10	15, 10	5, 10	40, 9

(a) What strategies survive the iterated deletion of strictly dominated strategies? For each iteration, specify the dominated strategy and the strategy that dominates it.

Note first that all of player 1's pure strategies are a best reply to some pure strategy of player 2 and therefore cannot be strictly dominated. Likewise, for player 2, all but strategy D are best responses to P. Thus, D is our only candidate. While D is not dominated by any pure strategy, it is dominated by a mixed strategy.

- D is strictly dominated by mixtures that put sufficient probability on B (e.g., $\frac{9}{10}B + \frac{1}{10}C$).
- Once D is eliminated, P is strictly dominated by N
- Then, C is dominated by mixtures of A and B (e.g., $\frac{1}{2}A + \frac{1}{2}B$)
- The remaining strategies are not dominated. Therefore, M, N, and O for player 1, and A and B for player 2 survive IDSDS.

Issues:

1.a.1 Strategy O is NOT strictly dominated in the reduced game, but only weakly dominated, and therefore cannot be eliminated by IDSDS.

1.a.2 Strategy C is not strictly dominated in the original game (it is weakly dominated) and thus it cannot be eliminated until after P is eliminated.

(b) What is the unique Nash equilibrium of this game?

Note that the wording of the question indicates that the game has a unique (obviously mixed) Nash equilibrium.
Player 2 must assign strictly positive probability to each of his remaining strategies (A and B). However, in the reduced game (following IDSDS) note that strategy O is weakly dominated and therefore will always achieve a lower expected payoff than M when player 2 puts any probability on B. Therefore, a mixed strategy equilibrium will use only strategies M and N for player 1.
Player 1 is indifferent between M and N when player 2 puts $\frac{7}{9}$ probability on A; Player 2 is indifferent between A and B when player 1 puts probability $\frac{1}{2}$ on each of M and N. Therefore, the Nash equilibrium is $\{\frac{1}{2}M + \frac{1}{2}N; \frac{7}{9}A + \frac{2}{9}B\}$

Issues:

1.b.1 Some students attempted to make player 1 indifferent between M, N, and O, found that there is no solution, and gave up. Note that a lack of solution implies only that player 1 can't be indifferent between *all three* pure strategies and therefore a mixed strategy must involve only two of the pure strategies.

1.b.2 It is possible to make player 1 indifferent between N and O, but remember that a mixed strategy equilibrium requires not only that a player is indifferent between the strategies that are played with strictly positive probability, but also that these strategies yield at least as much of an expected payoff as the strategies that receive zero probability. A proposed equilibrium using only N and O would fail the second criterion.

1.b.3 Showing that the game lacks a pure-strategy Nash equilibrium does *not* imply that the game doesn't have a Nash equilibrium.

1.b.4 For those that did put zero probability on strategy O, it is necessary to either show that it results in a weakly lower payoff than M and N (or note that it is weakly dominated).

(c) Imagine that this game is repeated twice with the outcome of the first stage observed by both players before the second stage. What is the maximum expected payoff Player 1 can earn in the first stage of any subgame perfect equilibrium of the twice-repeated game? Explain. `Since the stage game has a unique Nash equilibrium, any subgame perfect NE of a finite repetition of the stage game must involve playing the stage game NE in each period. Therefore, Player 1 must earn` $8 \times \frac{7}{9} + 2 \times \frac{2}{9} = 4 \times \frac{7}{9} + 16 \times \frac{2}{9} = \frac{60}{9}$ `in the first period.`

Issues:

1.c.1 It is correct that the maximum payoff is simply the equilibrium payoff of the stage game, but a number should be provided.

1.c.2 Note that failure to find an equilibrium in part b does not imply that there is no equilibrium in this game. As this is a finite game, a Nash equilibrium must exist.

1.c.3 In calculating player 1's payoffs, the expected payoff from M $(8\times\frac{7}{9}+2\times\frac{2}{9})$ is obviously equal to the expected payoff from N $(4\times\frac{7}{9}+16\times\frac{2}{9})$. Some students inexplicably summed the payoffs from M and N which produces double the expected payoffs.

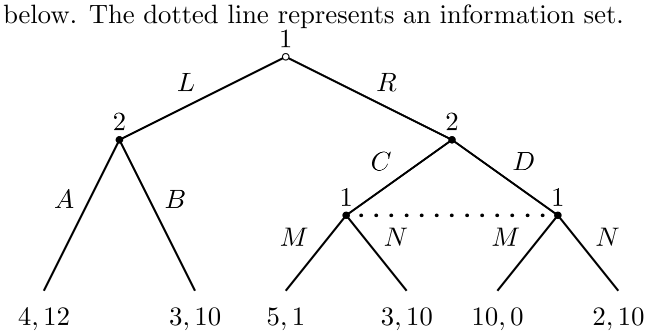
1.c.4 It is not possible to have any pure strategy equilibria in the first stage as the payoffs added to each first-stage outcome from the second stage subgame are all identical. Thus, the only first-stage outcome is the Nash equilibrium of the stage game.

1.c.5 Some provided the correct numerical answer but without the requisite explanation.

1.c.6 Avoid non-game-theoretic reasoning (like 'player 1 will play N because of the chance of earning the highest payoff').

1.c.7 Some devised alternate outcomes in stage 1 by having players threaten certain punishments in stage 2. Recall that any stage 2 play must be a Nash equilibrium, and thus threats that are not best replies are excluded by the definition of subgame perfection.

Question 2. Consider the game below. The dotted line represents an information set.



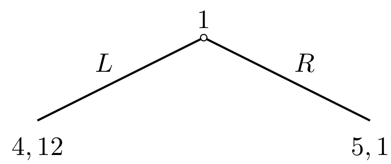
(a) List all subgame-perfect Nash equilibria.

There are two proper subgames, one beginning at each of player 2's decision nodes.

- The subgame on the left has player 2 choosing between A with a payoff of 12 or B with a payoff of 10. The Nash equilibrium of this subgame is $\{A\}$.
- The subgame on the right is shown below and has a unique equilibrium of $\{M, C\}$.

Player 1		Player 2: C	Player 2: D
	M	5, 1	10 0
	N	3, 10	2, 10

Thus, the game reduces to:



And player 1 selects R. Combining, the unique subgame-perfect Nash equilibrium is $\{R, M; A, C\}$

Issues:

- Some students found all Nash equilibria first which is of course acceptable but also somewhat more cumbersome.

2.1 Technically, the left subgame consists only of the pure strategies A and B. The strategy L does not exist in this subgame and thus should not be part of the description of a Nash equilibrium. Same for R in the right subgame.

2.2 Note that a strategy for each player consists of two actions and thus an equilibrium must specify two actions for each player.

2.3 Be careful in the right subgame. Even though player 2 goes ''first'' in the subgame (I put ''first'' in quotes as the subgame is simultaneous), her payoffs are listed second.

2.4 Don't forget after solving for the equilibrium in each proper subgame to still determine player 1's initial action.

2.5 Some identified $\{R, M; A, C\}$ as one SPNE but also concluded that $\{R, M; A, D\}$ is a SPNE. Note that the best response to M in the right subgame is C not D.

Question 3. Two firms compete in a differentiated-products Cournot market. In the first period, firm 1 selects a level of differentiation, $d \in [0,1]$. In the second period, both firms (after both observing d) simultaneously select quantities, $q_i \geq 0, \imath \in \{1,2\}$.

Inverse demand for firm i given its quantity choice, q_i, and that of the other firm, q_j, is given by:

$$p_i(q_i, q_j) = 1 - q_i - (1-d)q_j$$

And firm i's profit is given by $p_i q_i$ (there are no costs of production).

(a) Find the subgame perfect Nash equilibrium of this game.

Each $d \in [0,1]$ defines a subgame. In the second stage, taking d as given, each player solves:

$$\max_{q_i}(1 - q_i - (1-d)q_j)q_i$$

which yields (for an interior solution) the best reply function:

$$q_1(q_2; d) = \frac{1}{2} - \frac{1}{2}(1-d)q_2$$

and identically

$$q_2(q_1; d) = \frac{1}{2} - \frac{1}{2}(1-d)q_1$$

Substituting the second into the first yields the second-stage Nash equilibrium of $q_1(d) = q_2(d) = \frac{1}{3-d}$.

Note that this is hopefully intuitive since the monopoly solution is obtained when $d = 1$ and the Cournot duopoly solution is obtained when $d = 0$. The resulting price is $p_1 = p_2 = \frac{1}{3-d}$ and each firm's profit is given by $p_i q_i = \frac{1}{(3-d)^2}$

In the first period, firm 1 maximizes its (second period) profit with respect to d which occurs at $d = 1$. Therefore, the subgame-perfect Nash equilibrium is $\{1, \frac{1}{3-d}; \frac{1}{3-d}\}$.

The correct steps for solving this problem, given the timing, are:

(a) Solve for each firm's best response function $q_1(q_2; d)$ and $q_2(q_1; d)$ where d is a parameter.

(b) Solve the Nash equilibrium of the second stage game, $q_1(d)$ and $q_2(d)$. Note that the equilibrium consists of a number for each player which may depend on d.

(c) Determine firm 1's profit in the first period $\pi_1(d)$. Note that the profit should not depend explicitly on q_1 or q_2 as these are merely functions of d.

(d) Maximize firm 1's profit in the first period with respect to d.

Issues:

3.a.1 Many students made various calculation errors in computing the second-stage equilibrium. In some cases, the error was as simple as not realizing that the solution can be simplified by factoring the denominator; in others, a small arithmetic mistake early in the calculations led to some very ugly expressions. In general, this is not a concern as long as the correct steps of solving such a problem are followed.

3.a.2 In some cases, the arithmetic errors led to an equilibrium in which $q_1 \neq q_2$. Note that given the symmetry of the problem, any such answer generally would imply an identical equilibrium with the firms' strategies reversed. The fact that your equations do not allow for such a second solution should signal that an error was made.

3.a.3 Note that the timing involves both players simultaneously selecting quantities in the second stage. Some solved for $q_2(q_1)$ and then substituted this best response into player 1's profit function before maximizing with respect to q_1. This would only be valid if q_1 were selected prior to q_2.

3.a.4 The SPNE must include a value for d. Some omitted this first-stage maximization.

3.a.5 Some students calculated the best replies in the second period, but then plugged these into the profit equation and maximized with respect to d. One should first solve for the equilibrium of the second stage and then substitute out all second-period decision variables (q_1 and q_2) from the first period profit before maximizing with respect to d. Otherwise, you obtain a solution for d that is itself a function of q_1 which is backwards!

3.a.6 Due to arithmetic errors, some had the first-period profit *decreasing* in d and thus found an equilibrium in which $d = 0$. Given the nature of the problem and of the profit function, economic intuition should have you question this result.

3.a.7 Similarly, an answer that implies that firm 1 earns zero profits also should make one at least consider that the answer runs counter to intuition of the Cournot model. Does firm 1 not have any choice of d and then q_1 that can yield a positive profit?

3.a.8 Recall that the SPNE must include each firm's quantity in *every* subgame (for all d), and not only for the specific d obtained in period 1.

(b) If increasing d comes at a cost, what is the most that firm 1 would be willing to spend to increase d from 0 to 1? Explain intuitively.

All that this problem is asking you to do is evaluate firm 1's profit when $d = 0$ (in which case it is a duopolist with profit $\frac{1}{9}$) and compare it to firm 1's profit when $d = 1$ (in which case it is a monopolist with profit $\frac{1}{4}$). The most that firm 1 would be willing to pay to increased d from 0 to 1 is the added profit gained, $\frac{1}{4} - \frac{1}{9}$.

Given the arithmetic errors in part (a), the evaluation in this section was difficult for many of you. However, the key is understanding that you simply want to compare the profits at each value of d and take the difference.