Microeconomic Theory II — Spring 2023
Midterm Exam — Mikhael Shor

Solutions
Quick, Incomplete, and not Guaranteed

Question 1. Consider the normal form game below.

Player 1 \ Player 2	A	B	C	D
M	$9,1$	$7,0$	$7,2$	$7,4$
N	$7,1$	$5,3$	$1,2$	$9,1$
O	$9,2$	$9,0$	$9,4$	$7,2$
P	$3,3$	$1,2$	$1,2$	$8,3$

(a) What strategies survive the iterated deletion of strictly dominated strategies? Carefully explain each step.

- Note that all strategies except P are best responses to some pure strategy so cannot be strictly dominated. P is dominated by a mix of M, N, and O that puts probability $p \in (\frac{1}{2}, 1)$ on N.

- After eliminating P, A is dominated by C

- Next, M is dominated by any strategy that puts probability $p \in (\frac{3}{4}, 1)$ on O.

- Next, D is dominated by C.

- Next, N is dominated by O

- Next, B is dominated by C.

Therefore, the game is dominance solvable with only C and O surviving the IDSDS.

(b) Are any Nash equilibria of this game *not* trembling-hand perfect? Explain.

No. The game has a unique Nash equilibrium and it is trembling-hand perfect. There are two reasons.

- First, neither C nor O is weakly dominated, and in a two player game, THPE is equivalent to NE in strategies that are not weakly dominated.

- Second, a THPE must exist in this game; as the NE is unique, it must be THPE.

(c) What strategies are weakly dominated?

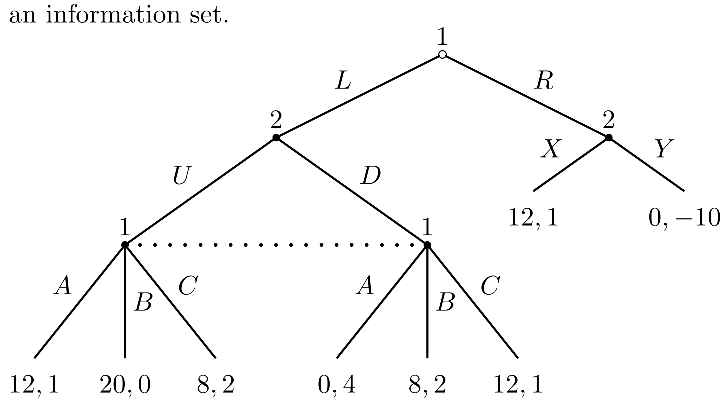
First note that N, O, B, C, and D are unique best replies to some pure strategy of the other player, so cannot be weakly dominated. That leaves only M, P, and A as candidates.

- P is strictly dominated (above) and therefore weakly dominated.

- M is weakly dominated by O

- A is weakly dominated by D

Question 2. Consider the extensive form game below. The dotted line represents an information set.



(a) List all subgame-perfect Nash equilibria.

The game has two proper subgames. In the subgame on the left:

		Player 2	
		U	D
	A	12, 1	0, 4
Player 1	B	20, 0	8, 2
	C	8, 2	12, 1

A is strictly dominated by B. There is no pure-strategy Nash equilibrium of this subgame, but there is a (unique) mixed-strategy Nash equilibrium of $\{\frac{1}{3}B + \frac{2}{3}C; \frac{1}{4}U + \frac{3}{4}D\}$ with expected payoffs of 11 for player 1 and $\frac{4}{3}$ for player 2.

In the subgame on the right, Player 2 selects X with payoffs of 12 and 1.

Thus, player 1 has a choice between L with payoffs of 11 or R with payoffs of 12 and selects R.

The SPNE is:

$\{R, \frac{1}{3}B + \frac{2}{3}C; X, \frac{1}{4}U + \frac{3}{4}D\}$ or $\{\frac{1}{3}BR + \frac{2}{3}CR; \frac{1}{4}XU + \frac{3}{4}XD\}$

(b) Does this game have a Nash equilibrium that is not subgame-perfect? Briefly explain.

Note that a Nash Equilibrium does not require a Nash in a subgame off the equilibrium path. So, for example $\{R, B; X, D\}$ is a Nash equilibrium.

If player 2 plays X, D, then $L, C; R, A; R, B$; or R, C all yield the maximum payoff so R, B is a best response. If player 1 plays R, B, player 2's best responses are X, U and X, D so X, D is a best response. Similarly, $\{R, A; X, D\}$ and $\{R, C; X, D\}$ are also pure-strategy Nash equilibria that are not subgame perfect. There also exist mixed strategy Nash equilibria that are not subgame perfect.

Note that you can't have a Nash equilibrium in which player 2 plays the pure strategy U in the left subgame since player 1's best response would then be L, B but then player 2's best response would not involve U. Intuitively, to make sure that the left subgame stays off the equilibrium path, player 1 should not be able to earn more than 12 so that R remains a best response.

Also note that the fact that the SPNE is unique doesn't say anything about whether there are other Nash equilibria, since the set of NE is at least as large as the set of SPNE.

Question 3. When a pharmaceutical drug maker advertises a new drug, the advertising can have positive spillover effects for competitors. Consider an industry consisting of two symmetric firms, 1 and 2, who are Cournot duopolists. Inverse demand is given by $P = A - q_1 - q_2$, where A is the amount of advertising and q_1 and q_2 are the firms' quantity choices.

Consider the following two-period game:

In the first period, firm 1 decides how much to invest in advertising, $A \geq 0$. The cost of advertising A is $\frac{2A^3}{81}$.

In the second period, firms observe A and the resulting inverse demand given by $P = A - q_1 - q_2$, and simultaneously select quantities, q_1 and q_2. There are no marginal costs. Second-period profits for firm i are given by Pq_i.

1. Describe the set of strategies for each player.

   For player 1, $\{A, q_1(A)\}, A \geq 0, q_1(A) \geq 0 \ \ \forall A$

   For player 2, $q_2(A) \geq 0 \ \ \forall A$

2. Find the subgame-perfect Nash equilibrium.

   The second period is Cournot with best responses given by $q_i(q_j; A) = \frac{A}{2} - \frac{1}{2}q_j$ and equilibrium of $q_1(A) = q_2(A) = \frac{1}{3}A$.

   In the first period, firm 1 maximizes $(A - q_1(A) - q_2(A))q_1(A) - \frac{2A^3}{81} = (A - \frac{1}{3}A - \frac{1}{3}A)\frac{1}{3}A - \frac{2A^3}{81} = \frac{1}{9}A^2 - \frac{2A^3}{81}$ which yields $A = 3$.

   Hence, the subgame-perfect Nash equilibrium is

   $\{A = 3, q_1(A) = \frac{1}{3}A; q_2(A) = \frac{1}{3}A\}$

   with an equilibrium *outcome* of $A = 3, q_1 = q_2 = 1$.

   Now imagine that instead of firm 1 selecting and paying for A, the firms form an advertising cooperative. Effectively, in the first period, the firms agree on a level of A that and share the cost of $\frac{2A^3}{81}$ equally between them. In the second period, as before, the firms simultaneously select quantities, q_1 and q_2.

3. What is the level of advertising, A, in the subgame-perfect Nash equilibrium?

   Either firm 1 maximizes the sum of the profits minus the cost of advertising or alternatively (and identically) its own profit minus half of the costs of advertising, which yields $A = 6$. Advertising increases since firm 2 no longer free-rides on the advertising of firm 1.