# Microeconomic Theory II
# Problem Set 1

This problem set covers rationalizable and strictly dominated strategies, best replies, and pure-strategy Nash equilibria.

The first three questions are in strategic form, with finite strategies.

1. Consider the following two-player game in strategic form:

| | L | C | R |
|---|---|---|---|
| T | 2,3 | 3,0 | 4,0 |
| U | 3,3 | 2,0 | 1,0 |
| M | 1,3 | 5,2 | 0,6 |
| B | 1,1 | 1,4 | 2,3 |

   Demonstrate that the game is dominance solvable (i.e., the iterative elimination of strictly dominated strategies results in a single outcome). For each eliminated strategy, specify what strategy strictly dominates it.

2. Consider the following two-player game in strategic form:

| | L | C | R |
|---|---|---|---|
| T | 4,5 | 3,0 | 0,2 |
| M | 5,2 | 2,1 | 1,0 |
| B | 0,0 | 2,8 | 4,2 |

   (a) What strategies survive the iterative elimination of strictly dominant strategies (recall the role of mixed strategies in determining what strategies are strictly dominated)?

   (b) What strategies are rationalizable (again, recall the importance of mixed strategies)?

   (c) What strategies are ruled out by the assumption of rationality alone (i.e., without the assumption of common knowledge)?

   (d) Find all pure-strategy equilibria.

3. Consider the following two-player game in strategic form:

| | L | R |
|---|---|---|
| U | 3,2 | 2,2 |
| M | 1,1 | 0,0 |
| B | 0,0 | 1,1 |

   In class, we discussed the iterated deletion of *strictly* dominated strategies, but did not apply this to weakly dominated ones. One reason is that a weakly dominated strategy may still be a best response, and thus consistent with rationality. A second is that elimination of weakly dominated strategies is not necessarily order independent.

   In the above, show that the iterated deletion of *weakly* dominated strategies can lead to different unique predictions depending on the order in which strategies are eliminated.

Questions 4 and 5 have continuous utility functions and continuous best reply functions.

4. Consider two players (1 and 2) who each determine a level of effort, $s_i \in [0, \infty)$, to devote to the production of a public good. Each player's utility is given by

$$u_i(s_1, s_2) = (s_1 + s_2)e^{-s_i}.$$

Both players select effort simultaneously.

   (a) What is player 1's best response function, $s_1(s_2)$?

   (b) Are any strategies never a best response?

   (c) Find all pure strategy Nash equilibria.

5. A group of $n$ students go to a restaurant. Looking at the menu, each discovers that more expensive menu items also have higher expected utility. Each student simultaneously selects a strategy, which is the price of her own meal. Each student has the utility function:

$$\sqrt{p} - \hat{p}$$

where $p$ is the menu price of the meal and $\hat{p}$ is the amount the student contributes towards the bill.

   (a) Assume that each student pays for her own meal (so that $\hat{p} = p$). What is the Nash equilibrium?

   (b) Assume that students agree prior to ordering to split the total check equally between them. What is the Nash equilibrium?

   (c) What happens when $n = 1$ and when $n \to \infty$ ?

Questions 6–8 are more challenging because best responses are not always unique and not continuous.

6. Hotelling model of political competition. There exists a continuum of voters, each with a favorite position on some issue, defined on the interval $[0, 1]$. For example, if the issue is illegal immigration, the position "1" can be an open borders policy, "0" can be a 500 foot fence, and intermediary values are more moderate positions. Two candidates simultaneously select their position, $s_i \in [0, 1]$. Each attracts the votes of all citizens whose favorite position is closer to her than to the position of the other candidate (with ties split evenly). A candidate's utility is 1 if she obtains strictly more votes than her rival, 1/2 if she ties, and 0 if she loses.
   (a) Assume that the distribution of voters' preferences is uniform on $[0, 1]$. What is the pure strategy equilibrium if the candidates simultaneously select their positions?
   (b) Assume that the distribution of voters' preferences is given by some generic distribution $F$. What is the pure strategy equilibrium?

7. Consider a game among $N$ players. Each simultaneously selects a number from the interval $[0, 100]$. The player closest to the mean of all selected numbers wins \$1, with ties resulting in an equal split.
   (a) If $N = 2$, what are the pure-strategy equilibria of this game?
   (b) If $N = 3$, what are the pure-strategy equilibria of this game?
   Try three different strategies for solving the game:
   (i) is it possible for players to select different numbers in equilibrium?
   (ii) is it possible to have an equilibrium where one player loses.
   (iii) Determine the equilibrium from each player's best responses

8. Consider a game among 3 players. Each simultaneously selects a number from the interval $[0, 100]$. The player *farthest* from the mean wins \$1, with ties resulting in an equal split. What are the pure strategy equilibria?
   [Hint] consider three possible cases:
   (a) all three players choose the same strategy,
   (b) two of the three players choose the same strategy, and
   (c) all players select different strategies.