Microeconomic Theory II
Problem Set 2

This problem set focuses on mixed strategy equilibria in normal-form games.

1. Two students are responsible for a joint assignment. A good grade requires costly effort on the part of both students. Each student can elect to invest effort (work) or not invest effort (shirk). The game in strategic form is:

	S	W
S	0,0	0,-c
W	-c,0	1-c,1-c

   where c, $0 < c < 1$, is the cost of effort.

   (a) Determine the game's two pure-strategy equilibria.

   (b) Determine the game's unique mixed-strategy equilibrium.

   (c) In the mixed strategy equilibrium, what happens to the probability of working as c increases?

   (d) What are each player's expected payoffs in the mixed strategy equilibrium? (Recall that, in a mixed strategy equilibrium, expected payoffs from each strategy are equal. The expected payoff in equilibrium is simply the expected payoff from either strategy). How do expected payoffs change as c increases?

2. Consider the following two-player game in strategic form:

	L	R
U	4,3	1,0
M	3,1	3,2
D	0,3	4,0

   This game has one pure strategy and two mixed-strategy equilibria.

   (a) What is the pure strategy equilibrium?

   (b) Is there a mixed strategy equilibrium in which Player 1 (the row player) uses all three strategies with strictly positive probabilities? (Try to equalize the payoffs from all three pure strategies. Does it work?)

   (c) Is there a mixed strategy equilibrium in which Player 1 assigns probability 0 to strategy U? (Recall that this requires that M and D yield the same expected payoff, and U yields a weakly lower expected payoff. Solve for the equilibrium *assuming* that U is never played and then check if U yields a strictly higher payoff).

   (d) Is there a mixed strategy equilibrium in which Player 1 assigns probability 0 to strategy M?

   (e) Is there a mixed strategy equilibrium in which Player 1 assigns probability 0 to strategy D?

3. Consider the following two-player game in strategic form:

	L	M	R
A	2,2	2,2	2,2
B	3,3	0,2	0,0
C	0,0	3,2	0,3

This game will demonstrate several methods for ruling out possible mixed-strategy equilibria.

(a) What are the pure strategy equilibria?

(b) Show that there does not exist an equilibrium in which Player 1 (the row player) assigns strictly positive probability to A, to B, and to C.

(c) Show that there does not exist an equilibrium in which Player 1 (the row player) assigns strictly positive probability only to A and to B. [Hint: what would Player 2 do?]

(d) Show that there does not exist an equilibrium in which Player 1 (the row player) assigns strictly positive probability only to A and to C.

(e) Show that there does not exist an equilibrium in which Player 1 (the row player) assigns strictly positive probability only to B and to C.

(f) The above items demonstrate that there is no equilibrium in which Player 1 uses a mixed strategy that assigns positive probability to more than one pure strategy. Find all equilibria in which Player 2 uses a mixed strategy that assigns positive probability to more than one pure strategy.

4. Consider the following two-player zero-sum game in strategic form:

	L	C	R
T	-1,1	1,-1	1,-1
M	2,-2	-2,2	2,-2
B	3,-3	3,-3	-3,3

(a) Show that there does not exist a pure-strategy Nash equilibrium

(b) Find the mixed strategy equilibrium that assigns strictly positive probability to every strategy.

(c) What is each player's expected equilibrium payoff?

(d) Next, we want to show that the above equilibrium is unique:

   i. Show that there does not exist an equilibrium in which Player 2 (the column player) assigns probability 0 to strategy L

   ii. Show that there does not exist an equilibrium in which Player 2 (the column player) assigns probability 0 to strategy C

   iii. Show that there does not exist an equilibrium in which Player 2 (the column player) assigns probability 0 to strategy R

5. The 1000 workers of Farm, Inc., all live in FarmVille, a town 20 miles from the factory. There are two roads from home to work. Wide Road is sufficiently wide that traffic can travel consistently at 40 mph, regardless of the number of cars. Thus, the commute on Wide Road takes 30 minutes. Narrow Road has a speed limit of 60 mph, but actual speed depends on congestion. Specifically, the commute time on Narrow Road $= 20 + \frac{m}{30}$ where m is the number of FarmVille drivers that take Narrow Road. The 1000 workers each simultaneously and independently select a strategy of either Wide Road or Narrow Road. Each worker's utility is given by

$$X - commute\ \ time$$

where X is some constant.

   (a) What are the pure strategy equilibria of this game?

   (b) There also exist mixed strategy equilibria of this game, where workers randomize between Wide Road and Narrow Road. In equilibrium, what is the expected commute time on Narrow Road?

6. Consider the problem of whether to stop and help somebody whose car has broken down on the side of the road. Imagine that each of N drivers on the road must decide whether to stop and help or to go past. Each driver receives a utility of 1 if *someone* stops to help. Each driver has a cost of stopping given by $c < 1$.

   (a) If $N = 1$, what would the single driver do?

   (b) For $N > 1$, determine the pure-strategy Nash equilibria.

   (c) Now find a symmetric Nash equilibrium, in which each driver stops with the same probability.

   (d) How does the likelihood of a given person stopping vary with N?.

   (e) How does the likelihood that *someone* stops vary with N? Are you more likely to get help if your car breaks down on a more- or less-busy road?