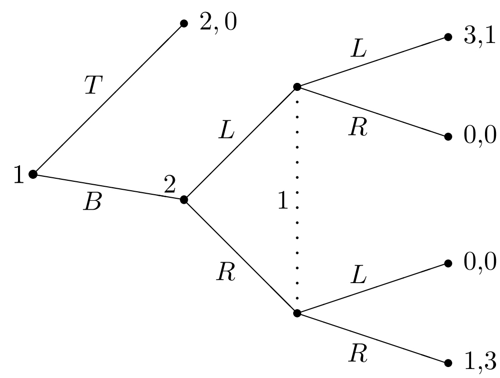
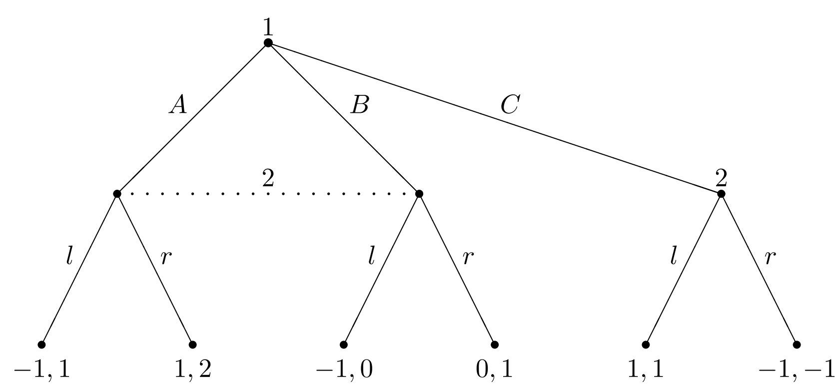
Microeconomic Theory II
Problem Set 3

This problem set focuses on subgame-perfect Nash equilibria and extensive form games.

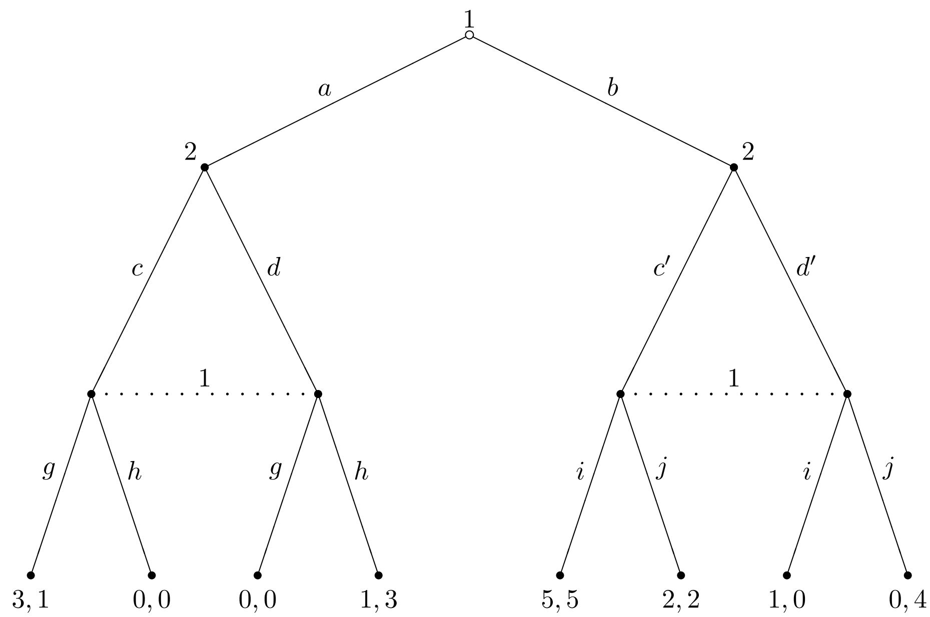
1. Consider the game in extensive form below.
   (a) Find the set of pure-strategy Nash equilibria.
   (b) Find the set of pure-strategy subgame-perfect Nash equilibria.
   (c) Are there any mixed-strategy Nash and subgame-perfect Nash equilibria?



2. Find all Nash and subgame-perfect Nash equilibria of the game below



3. Find all pure-strategy subgame-perfect Nash equilibria of the game below



4. Revisit a question from the first problem set. Consider two players (1 and 2) who each determine a level of effort, $s_i \in [0, \infty)$, to devote to the production of a public good. Each player's utility is given by

$$u_i(s_1, s_2) = (s_1 + s_2)e^{-s_i}.$$

Player 1 selects effort first and player 2 selects effort after observing player 1's decision. Determine the subgame-perfect Nash equilibrium.

5. Robinson Crusoe is no longer alone on his island! Man Friday has formed a government with the power to tax Crusoe to fund government spending, g. Crusoe consumes in two periods and has an initial endowment of capital, k_1. In the first period, Crusoe selects a level of consumption, c_1. In the second period, Friday selects a tax level on capital, t.

   The production function is linear is so that capital can be preserved or transformed one-for-one into consumption:

$$k_2 = k_1 - c_1$$

   Additionally,

$$k_2 = g + c_2, g = tk_2$$

   Crusoe's utility function is given by: $\ln c_1 + \delta \ln c_2$

   Friday's utility function is given by: $\ln g + \beta \ln c_2$

(a) Rewrite each player's utility function so that it depends only on strategies and exogenous parameters

(b) What is the subgame perfect Nash equilibrium of this game?

(c) What is each player's utility in equilibrium?

(d) Imagine that the order of decisions were reversed, so that first Friday announced a tax rate, and then Crusoe selected first-period consumption. How does the subgame-perfect equilibrium and players' utilities compare to your answer above?

6. Five pirates must decide how to split 100 indivisible pieces of silver. The pirates are ranked in terms of seniority, with Pirate 1 the least senior and Pirate 5 the most senior. The well-known standard pirate procedure for allocating pieces of silver works as follows:

   (a) First, Pirate 1 proposes a split of the 100 pieces of silver.

   (b) Pirates vote on whether or not to accept the division.

   (c) If a majority of pirates (strictly more than half, and including the vote of Pirate 1) vote for the split, the pieces of silver are divided as proposed and the game ends.

   (d) If a majority does not vote for the split, the pirate is thrown off the ship to his death. Then, Pirate 2 proposes a split of the 100 pieces of silver, and again a strict majority of the remaining pirates must vote for the split or else Pirate 2 is killed, and so on.

   Pirates have lexicographic preferences. Each pirate's utility is strictly monotonically increasing in the number of pieces of silver he receives (where death is equivalent to 0 pieces of silver). Further, for a given allocation of silver, a pirate prefers that other pirates die rather than live. That is, if a pirate is indifferent in terms of expected silver allocation from accepting or rejecting a split, he will always vote to reject. It is well-known that pirates are brilliant, rational game theorists, and that this is common knowledge among pirates.

   (a) What is the subgame-perfect allocation of silver?

   (b) What if the number of pirates is greater than 5? What pattern emerges if you consider 6 pirates, then 7, . . . ?

   (c) Consider again five pirates, but imagine that a division is accepted if at least half of remaining pirates vote to accept (i.e., they don't need a strict majority). How does this change your answer to (a)?

7. Consider a repeated game in which the following stage game is played twice (with the outcome of the first stage observed prior to the second stage):

		Player 2		
		X	Y	Z
Player 1	A	8, 8	12, 2	4, 8
	B	2, 12	10, 10	3, 4
	C	1, 1	3, 3	5, 5

(a) Is there a subgame-perfect equilibrium in which the players play (B, Y) in the second stage?

(b) Is there a subgame-perfect equilibrium in which the players play (B, Y) in the first stage?