Microeconomic Theory II
Problem Set 4

This problem set focuses on oligopoly models.

1. Consider the n-player Cournot model with linear demand and cost functions:

$$P(Q) = \max\{0, a - bQ\}$$
$$c(q_i) = cq_i$$

   (a) Find the unique symmetric Nash equilibrium

   (b) We showed in class that this game is dominance-solvable when $n = 2$. Show that this is not the case when $n = 3$.

2. Consider a Stackelberg model with $P(Q) = 1 - Q$ and marginal cost, $c = 0$. Assume that 1 firm (the leader) sets its quantity in the first period. Then, n firms (followers) set their quantities simultaneously in the second period after observing the leaders quantity.

   (a) Determine the subgame-perfect Nash equilibrium. (Note that the second stage is equivalent to a Cournot market like you solved above).

   (b) How does the leader's quantity decision change with the number of followers?

   (c) Now imagine that instead of 1 leader, there are m leaders who simultaneously set quantities in the first period. Then, the n followers set their quantities simultaneously in the second period. Determine the subgame-perfect Nash equilibrium (you may assume that the equilibrium in each stage is symmetric).

3. Consider an n-firm Bertrand model with demand given by

$$Q(P) = \max\{0, 1 - P\}$$

where $P = \min\{p_1, \ldots, p_n\}$ is the lowest price charged by any firm.

Firms are homogeneous and consumers purchase at the lowest available price. Therefore, profit is given by:

$$\begin{array}{ll} 0 & \text{if } p_i > P \\ P \times Q(P)/t & \text{if } p_i = P \end{array}$$

where t is the total number of firms that charge P.

(a) Find the unique Nash equilibrium of this game.

(b) What price, if charged by each firm, maximizes total profit? What is each firm's resulting profit?

(c) Assume all other firms are charging the price in (b)? What is a firm's best response? What profit does this firm earn?

(d) Assume that the above game is played repeatedly and infinitely. If the firms use the grim trigger strategy (charging the price in (b) in every period as long as every firm has charged the price in (b) in every previous period), what has to be true of the interest rate for this to be a subgame-perfect equilibrium (i.e., for collusion to be sustainable)?

(e) What happens to your answer in (d) as the number of firms increases? Is this intuitive?

(f) *** Show that the answer you obtained in (d) is true for *every* reasonable demand function.