Microeconomic Theory II
Problem Set 6

This problem set provides an example of a principal-agent model.

1. In class, we considered a numerical example of a simple two-effort, two-outcome principal agent model. Here, we reconsider this example for general values. A risk-neutral agent can either put in low or high effort. The probability of project success is given by p^l if the effort level is low, and by p^h if the effort level is high (in class, these were 0.6 and 0.8). The principal receives revenues of v if the project succeeds and of 0 if it does not (in class, we considered $v = 600$). The agent has a cost of low effort of c_l and of high effort c_h (in class, these were 100 and 150).

   (a) Determine the optimal contract that induces low effort on the part of the agent.

   (b) Determine the optimal contract that induces high effort on the part of the agent.

   (c) When does the principal prefer to induce high effort?

2. Consider a principal-agent problem. An agent puts in one of two effort levels, $e \in \{l, h\} \equiv \{0, 5\}$. There are three possible outcomes, $x \in \{x_1, x_2, x_3\} \equiv \{0, 100, 400\}$. Let p_i^j be the probability of outcome i with effort level j. The principal is risk-neutral with utility function $E[x - w(x)]$. The agent's utility function is given by $U(w, e) = u(w) - v(e)$.

   The probabilities are given by:

   $$p_1^l = .6, p_2^l = .3, p_3^l = .1$$
   $$p_1^h = .1, p_2^h = .3, p_3^h = .6$$

   (a) Assume that $u(w) = w$, $v(e) = e^2$, and the agent has an outside option that yields a utility of 81 (the utility of not taking the job is 81). Derive an optimal incentive contract (i.e., $w_1 \equiv w(x_1)$, $w_2 \equiv w(x_2)$, and $w_3 \equiv w(x_3)$).

   (b) If e were observable, so that the principal could contract directly on e, how would the principal's profit compare to the profit he obtains under the previous answer?

   (c) Now assume that $u(w) = w^{\frac{1}{2}}$, $v(e) = e$, and the agent has an outside option that yields a utility of 9. Derive the optimal incentive contract.

   (d) If e were observable, how would the principal's profit compare to the profit he obtains under the previous effort.

   (e) In (c), assume that a minimum wage law requires that $w(x) \geq 81 \forall x$. Derive the optimal incentive contract. [Note that individual rationality might not be binding.] What is the principal's profit?

   (f) In (c), assume that a minimum wage law requires that $w(x) \geq 100 \forall x$. Derive the optimal incentive contract. What is the principal's profit?