On the Competitive Effects of Bidding Syndicates

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Recommended Citation
DOI: 10.1515/1935-1682.2398

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Vlad Mares and Mikhael Shor

Abstract

Firms commonly form syndicates to bid jointly for financial assets. Recently, this practice has come under legal scrutiny motivated by models which suggest syndicates are anti-competitive. These models do not account for two important features of financial markets: bidders’ value estimates are likely to be correlated, and complicated mechanisms known to be optimal in such settings are usually eschewed in favor of simpler auction formats. We show that these features make it possible for syndicate bidding to generate higher revenues for the auctioneer than bidding among independent firms, even when syndicates are asymmetric or lead to a highly concentrated market. This occurs because syndication can make the industry more suitable to the simple auction format in use. We identify conditions under which syndicates are pro-competitive and discuss the implications for antitrust.

KEYWORDS: financial syndicates, auctions, antitrust, joint bidding

*The authors would like to thank Luke Froeb, Ron Harstad, Paul Klemperer, Dan Levin, Ron Masulis, John Morgan, David Parsley, the editor, and an anonymous referee for valuable comments and suggestions.
1 Introduction

In 2006, the Department of Justice (DOJ) opened an investigation into the way private equity firms bid for takeover targets. At issue were “club deals” in which several private equity firms form syndicates to submit a joint bid. Concurrently, shareholders of takeover targets sought damages in private civil lawsuits. Naming major private equity firms as defendants, shareholders contended that they were “deprived of the full economic value of their holdings” as club deals dampened competitive pressures that would exist if firms bid separately.\(^1\) Initial public offering syndicates were sued on similar grounds.\(^2\) According to Securities and Exchange Commission (SEC) commissioner Paul Atkins, “This suit seeks to trump the securities laws with the antitrust laws” arguing that an unfavorable ruling “could devastate America’s process of capital formation, wreak unprecedented havoc” and jeopardize “the stability in our capital markets” (Atkins, 2006).

The alarm on the part of the SEC reflects a significant difference in perspective between the SEC and the DOJ. Syndicate bidding is pervasive in financial markets, from angel investors (Sohl, 1999, May, 2002) and venture capital firms (Lerner, 1994) to underwriting of primary equity (Corwin and Schultz, 2005) and commercial lending (Dennis and Mullineaux, 2000). A majority of recent venture capital funding and nearly half of large private acquisitions are by syndicates rather than single firms (Lai, 2005, Berman and Sender, 2006). Conversely, “syndication” among bidders is often seen as merely a euphemism expression for collusion by the DOJ, which pursues more criminal convictions for bid-rigging than for all other market conspiracies combined (Froeb and Shor, 2005).

After an equivocal decision by the Supreme Court, lower courts have diverged on their handling of private suits against syndicates.\(^3\) One federal judge dismissed a case alleging that shareholders of a company acquired through joint bidding were denied economic profit.\(^4\) Months later, another federal judge allowed

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\(^3\)The Supreme Court’s ruling in Credit Suisse (ibid.) sidestepped the issue of primacy between securities and antitrust laws, holding that the argument was not “preserved” as it was not sufficiently argued in the lower courts. Instead, the Court opted for a “fact specific” decision (Baker and Ostrau, 2007). This leaves open the question of whether the SEC’s regulatory framework is sufficient, effectively immunizing all financial syndicates from antitrust review, or, conversely, if financial institutions will be judged by the same standards as candy stores and steel mills. For example, a U.S. Court of Appeals dismissed a case against prime lenders due to conflicts with SEC regulations (Electronic Trading Group v. Banc of America Securities 588 F.3d 128, 2nd Cir. 2009) while a district court allowed a case against municipal mortgage lenders to proceed, finding no regulatory conflict (Hinds County v. Wachovia Bank, 620 F.Supp.2d 514, S.D.N.Y. 2009).
a similar class-action case to proceed, rejecting the argument that club deals are immune from antitrust oversight. The still-ongoing suit has recently been broadened to include many of the largest corporate buyouts in history (Lattman, 2011). That one judge concluded that club bidding can be pro-competitive while another found sufficient evidence of anticompetitive effects signals that the issue has moved from the purvey of attorneys to economists.

The portfolio diversification justification often put forward by proponents of syndication interprets syndicates as vehicles for capitalization and risk sharing (Wilson, 1968, Chowdhry and Nanda, 1996, Lockett and Wright, 1999, Stuart and Sorensen, 2001). Proponents often point to deals that could not have happened in the absence of joint bidding. Yet, capitalization and risk-sharing cannot explain syndicated bidding for small deals, and are becoming less persuasive in general as financial firms grow in size. For example, Chen and Ritter (2000) note the increasing size of modern financial firms, arguing “Today, there is little reason to form a syndicate to perform the traditional economic roles of risk sharing, distribution, and meeting capital requirements” (p. 1120). This raises the question of why syndicates persist in the current environment. Some have proposed competition-reducing motivations as one possible answer. A noted antitrust attorney warned “If a bidder or group was able to bid on its own—but to avoid competition joined with other bidders capable of bidding on their own—that could be viewed as unlawful.”

In this study, we investigate the competitive effects on prices when bidders form syndicates rather than participate individually. To fix ideas, suppose that six private equity firms all show interest in acquiring a corporation. All six are sufficiently well capitalized and risk neutral, and have equivalent management abilities. That is, each is capable of bidding on its own, and the value of the corporation will not depend on the identity of the acquiring firm. However, firms do not know this value with certainty. Place yourself in the position of the target corporation’s major

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6These arguments have also gained favor in U.S. courts, dating back at least to an 1854 Supreme Court decision (Kearney v. Taylor, 56 S. Ct. 494), and forming the basis of decisions to this day (e.g., Love v. Basque Cartel, 873 F.Supp 563, D. Wyo. 1995).

7Lauren Albert (partner at Axinn, Veltrop, & Harkrider), cited in Berman and Sender (2006). Another attorney attributed the aforementioned legal investigations to a theory “that the formation of consortiums dampens competition in auctions because sponsors who would otherwise be bidding against each other team up to jointly bid and drive down the sale prices” (Schwartzman, 2006, p. 99). Other motivations forwarded for syndication include relationship networks (Hochberg, Ljungqvist, and Lu, 2007), monitoring to avoid moral hazard (Pichler and Wilhelm, 2001), and reducing coordination costs (Wright and Robbie, 1998). Syndicates, like mergers, can also lead to cost-reducing synergies, which manifest in more competitive prices (Sullivan, 2002).
shareholder. To maximize the price the acquiring firm pays for your shares, would you prefer that each bid separately, or that the firms organize themselves into two syndicates, with each syndicate submitting only one bid? 

Conventional models find two effects of syndication (Bikhchandani and Riley, 1991, Bulow and Klemperer, 2002, Mares and Shor, 2008). First, syndicates bid more aggressively than individuals as syndicates pool members’ information to form better value estimates. Second, syndicates nevertheless reduce the resulting price as the loss of competition leads to fewer overall bids. The competition effect dominates the information pooling effect even when the seller has responds to syndicated bidding by optimally altering the auction mechanism (Mares and Shor, 2009). These models confirm a common regulatory assumption that syndication is generally anticompetitive.

The results of these models are predicated on two assumptions. First, it is assumed that bidders’ estimates of value are uncorrelated. Second, the seller uses an optimal, revenue-maximizing mechanism. Both assumptions are unrealistic descriptors of financial syndicates. While the first assumption that bidders’ signals—their private information—are independent is very common in the auction theory literature, financial analysts’ signals are usually correlated with the realized value and with each other (Brown and Richardson, 1987, Barron, Byard, Kile, and Riedl, 2002). From a mechanism design perspective, a series of well-established results show that even arbitrarily small correlation among signals allows the auctioneer to design a mechanism that extracts full surplus (Crémer and McLean, 1985, 1988, McAfee and Reny, 1992). Under this optimal mechanism, bidders earn zero profits whether or not they form syndicates, making moot the question of competitive effects. Yet, these optimal mechanisms are quite complex, exhibit significant implementation challenges, are very sensitive to small variations in the model, and have never been adopted in practice. Crémer and McLean (1988) note that these mechanisms place unrealistic requirements on the auctioneer and bidders, suggesting that “less profitable but vastly simpler auctions [are] used in practice” (p. 1254). The question of how syndicates impact typical auction mechanisms (such as ascending and sealed bid auctions) despite their suboptimality is the subject of this paper.

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8 In another context, would a firm considering an equity offering realize lower or higher underwriting fees if it limited the power of competing investment banks to form underwriting syndicates? 

9 Full surplus extraction requires affiliation, a stronger concept than correlation, loosely defined as “local positive correlation everywhere” (Klemperer, 2004, p. 48).

10 These mechanisms require risk-neutrality and unlimited budgets and liability as they may require firms to risk arbitrarily large amounts of capital. Furthermore, bidders’ participation in full-surplus extracting mechanisms is in question whenever small information acquisition and bidding costs are present (Harstad, 2005). A theoretical critique is that full-surplus extraction is not a generic result (Neeman, 2004, Heifetz and Neeman, 2006).
When values are correlated and the auctioneer uses a typical auction mechanism, our results identify a third effect of syndication, in addition to information pooling and competition. Syndicates change the industry’s information profile—the distribution of available information among active bidders. This change can move the industry to a profile more suitable to the auction format in use. A typical auction’s revenue could be far from optimal in the absence of syndicates but the same auction format could perform substantially closer to the optimum when information is concentrated among fewer, but larger syndicates. We term this phenomenon the suboptimal design effect. Under identified conditions, the suboptimal design effect dominates the loss of competition, causing syndication to raise the auctioneer’s revenue. In these cases, the objectives of antitrust and securities regulation with respect to syndicated bidding need not be in direct conflict.

The effect of syndication depends on the selling format used (e.g., sealed bid or open auction) and the relationship between bidders’ information and the asset’s actual value. Our results demonstrate that syndication can be pro-competitive only in sealed-bid auctions, and not, for example, in English auctions in which bidders update their beliefs about the asset’s value upon observing other bidders drop out. Our results focus only on the competitive bidding of individual firms versus syndicates. Where pro-competitive effects are found, these must still be weighed against other potential dangers of syndicates. These include the systemic financial risk created by syndicate members holding similar portfolios (Cai, Saunders, and Steffen, 2011) and the greater likelihood of collusion among fewer active bidders (Klemperer, 2002).

Whether syndicates increase or reduce competition is not merely a regulatory question. For the owners of financial assets, firms with equity offerings, or potential takeover targets, allowing or prohibiting syndicate bidding is a strategic decision with serious consequences. It is not uncommon for firms to stipulate limitations on syndicates as part of their confidentiality agreements governing the disclosure of financial information.11 By precluding syndicates, some firms may be costing their shareholders potential gains.

The next two sections introduces the model and demonstrate that syndicates can raise or lower the resulting price depending on the auction format and the value function. A value function maps individual bidders’ estimates of the object’s value into a common estimate.12 Robustness of our result is examined in three dimen-

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11 Typical language to this effect in confidentiality agreements reads “For a period of one year, [the potential bidder] ... will not participate in or encourage the formation of any partnership, syndicate, or other group which seeks to acquire ownership of any Securities in or which seeks to affect control of [the target].” The SEC online EDGAR database contains multiple examples.

12 For example, if one were privy to the estimates of all bidders, would one rely more on the minimum, the maximum, the average, or some other functional form, to derive an overall estimate?
sions. First, variations in the value function are examined. The suboptimal design effect is sufficiently robust, inducing pro-competitive effects of syndicates under a broad class of value functions. Second, our examination considers asymmetry of syndicates’ information profiles. Certainly, 500 major firms forming one syndicate against a lone small firm may differ from two syndicates of 250 firms each. Third, our results carry over to uniform-price, multi-unit auctions. These models are loosely descriptive of bond auctions and of competitive underwriter selection for equity offerings (Parlour and Rajan, 2005). The conclusion discusses implications, limitations, and a simple screen in the form of a thought experiment which portends the effects of syndication in specific environments.

2 Model

A total of \( n \) firms participate in an auction for an asset with a common value \( v \). Firm \( i \) privately observes a signal \( s_i \). Denote by \( s \) the vector of signal realizations. The value of the asset is given by \( v(w, s) \), which is a function of the signals and a value parameter, \( w \). Prior beliefs about \( w \) are uniform on \([w, \bar{w}]\). This assumption is intended to limit priors to the least informative case. Private signals are conditionally independent and uniformly distributed on \([w - \theta, w + \theta]\). The spread parameter, \( \theta > 0 \), controls the precision of individual signals. This structure ensures that signals are affiliated with each other and with the underlying value. The variance of individual signals is unaffected by the realization of \( w \), leaving the precision of private information independent of the true value.

We compare equilibrium bids and revenues under two scenarios, one without syndicates, in which each of the \( n \) firms places a competitive bid observing only its own signal, and one with a highly-concentrated information profile consisting of two syndicates with \( n \) signals between them. Each syndicate, privy to all of the signals of its members, places a single bid. The polar case of firms consolidating into two syndicates is adopted for three reasons. First, it is the most concentrated information profile that still allows for competition. Therefore, any pro-competitive effects from forming only two syndicates are especially stark. Second, analytical solutions are possible in these cases, allowing for direct comparisons. Lastly, in intermediate cases, with multiple bidders each with multiple signals, existence of equilibria is not guaranteed (Jackson, 2009).

\( ^{13} \)In the text, we consider only cases where signals are in \([\underline{w} + \theta, \bar{w} - \theta]\). Signals in this interior range are unbiased, \( E[w|s_i] = s_i \). Effectively, we assume that the distribution of \( w \) has a diffuse prior; for any signal a bidder receives, the bidder does not know if that signal is higher or lower than that of other bidders. This can be operationalized by assuming that \( \underline{w} \to -\infty \) and \( \bar{w} \to \infty \) or, as we do here, by assuming that signals are sufficiently in the interior.
We consider two formulations of the value function. The most prevalent model of a common value auction with affiliated signals serves as our benchmark case:

\[ v(w, s) = w \]  

(1)

Nature draws some value from an uninformative prior and then provides each bidder with a conditionally independent signal centered on this value.\(^\text{14}\) Later, we consider a more general value function to isolate the source of competitive effects.

We consider both a second-price sealed-bid auction and an open ascending (English) auction. In the second-price auction, the highest bidder wins the asset and pays a price equal to the second-highest bid. In the English auction, prices continuously rise with bidders either indicating their willingness to pay the current price or choosing to drop out publicly and irrevocably. A bidding strategy maps one’s signals and the history of bidders who have dropped out into a price at which the bidder would drop out. In our context, English auctions weakly revenue-dominate second-price auctions which, in turn, dominate first-price auctions (Milgrom and Weber, 1982a).\(^\text{15}\) In private-value settings, where bidders have idiosyncratic but known values for the asset, the English and second-price auctions are strategically equivalent. With correlated signals, the English auction provides additional information to bidders as they may draw inferences from other bidders’ dropping out of the bidding. We derive the equilibria and revenues when firms do and when they do not form syndicates.

### 3 The Benchmark Model

First consider the value function given in Equation (1), where \(v(w, s) = w\). Because the signal each firm receives is an unbiased estimate of the object’s value, the expected value of the object equals one’s signal, \(E[v|s_i] = s_i\). However, a bidder will not bid this amount in equilibrium as it does not account for the winner’s curse, or the adverse selection inherent in selecting the auction’s winner. The bidder who wins learns that his signal is the highest of all bidders, and thus should condition his bid on this fact, bidding lower than one’s signal.

\(^{14}\)For example, this model has been used to study endogenous bidder entry (Harstad, 1990), properties of rationing or “lottery” auctions (Harstad and Bordley, 1996), the role of experience in overcoming the winner’s curse (Kagel and Richard, 2001), and the rationing of oversubscribed IPOs (Parlour and Rajan, 2005), and most experimental treatments of common-value auctions. Its equilibrium properties when there are no syndicates are well known (e.g., Klemperer, 2004).

\(^{15}\)First price auctions are particularly difficult to analyze in our set-up with multi-dimensional signals, especially under asymmetry. Numerical computations in DeBrock and Smith (1983) show that they yield qualitatively similar results to those we find in the second-price format.
Claim 1. In a second-price auction, the equilibrium bid in the benchmark model without syndicates is given by

\[ b(s) = s - \left( \frac{n-2}{n} \right) \theta. \]  

Claim 2. In an English auction, the equilibrium drop-out point in the benchmark model without syndicates is given by

\[ b(s) = \begin{cases} 
  s & \text{if no one has yet dropped out} \\
  \frac{1}{2}s + \frac{1}{2}s_{\min} & \text{otherwise}
\end{cases} \]  

where \( s_{\min} \) is the price at which the first bidder dropped out.

The expected value of the asset conditional on all signals (for an omniscient bidder) is \( E[v|s] = \frac{1}{2} \min\{s_1, \ldots, s_n\} + \frac{1}{2} \max\{s_1, \ldots, s_n\} \). In a second-price auction, no winner’s curse correction is required when there are precisely two bidders (Milgrom and Weber, 1982a), so one simply bids the expected value of \( v \) given one’s signal. For two syndicates, a similar result applies, in that each syndicate bids the expectation of \( v \) given all signals of its member firms. The English auction with two syndicates ends as soon as the first bidder drops out. Since a syndicate cannot glean any insight from the drop-out behavior of its only rival, bids in the English and second-price auctions are identical.

Claim 3. In second-price and English auctions with two syndicates, the equilibrium bid in the benchmark model of a syndicate with \( m \) member firms and signals \( s_1, \ldots, s_m \) is given by

\[ b(s_1, \ldots, s_m) = \frac{1}{2} \min\{s_1, \ldots, s_m\} + \frac{1}{2} \max\{s_1, \ldots, s_m\}. \]

Whether or not syndicates hurt competition depends on which information profile, individual bidders or syndicates, results in higher revenues. First, consider competition among \( n \) firms compared with competition among two symmetric syndicates, each with \( n/2 \) members.

Proposition 1.

1. In an English auction, the expected revenue from two symmetric syndicates is lower than the expected revenue in the absence of syndicates.
2. In a second-price auction, the expected revenue from two symmetric syndicates is higher than the expected revenue in the absence of syndicates.
Syndicates can have pro-competitive effects in a sealed bid auction but not in an open auction. These auction formats differ inherently in their ability to inform bidders about the asset’s value. An auction with observable exit reduces bidder uncertainty as bidders update their estimates each time a lower bidder drops out of the auction (Crampton, 1998). The final price reflects not only the private information of the price-setting bidder but the inferred information of all bidders below him. Allowing for syndicates in this context does not improve on the information available to the price-setting bidder, but does reduce competition, leading to lower prices. Auctions in which bids are submitted independently and without knowledge of others’ bids provide much less information about the object’s value. In these cases, syndicates substitute for the information role of an open auction. By combining the signals of its constituent members, the pooling of information within syndicates offsets the lack of competition among syndicates.

In light of recent results on independent signals, which hold that syndicates always reduce revenue (e.g., Waehrer and Perry, 2003, Mares and Shor, 2009), our result may seem counter-intuitive. Past results show that under optimal mechanisms, the competition effect (which reduces the number of bidders) dominates the information pooling effect (which provides syndicates with better information than individual bidders). To understand our result, we consider these effects in turn.

Without syndicates, the winner’s curse adjustment in sealed-bid auctions pushes bids significantly below one’s signal (Eq. 2). Increased competition drives each bid down towards $s - \theta$ but also drives the highest signals to the upper bound of the distribution, $v + \theta$. Through the competition effect, the price converges to $v$ asymptotically, effectively aggregating bidders’ disparate information.

With two syndicates, an increase in the number of constituent bidders does not intensify competition, but does improve a syndicate’s information. The second-price auction allows a syndicate to bid as if no winner’s curse problem exists (Eq. 4). An increase in a syndicate’s size, $m$, quickly leads to tight estimates of the asset’s value. Therefore, syndicates’ information rents disappear rapidly since equally well-informed bidders are likely to place very similar bids.

Another effect of syndication follows from the fact that a syndicate’s equilibrium bid incorporates both the highest and the lowest signal. Tying price to multiple sources of information generally improves auction revenues (Parlour and Rajan, 2005, Mezzetti and Tsetlin, 2009). The lowest signals contain as much information about the true value of the asset as do the highest signals, yet in the absence of syndicates, only the two highest signals (possessed by the winning and price-setting bidders) are incorporated into the price. In sealed bid auctions, these effects tip the balance in favor of syndicated bidding despite a loss of competition.

The revenue-improving role of syndicates in sealed bid auctions does not require symmetry, as the following proposition demonstrates.
Proposition 2. Consider $n \geq 3$. A second-price auction with two asymmetric syndicates (of sizes $m$ and $n - m$) leads to higher expected revenue than a second-price auction without syndicates as long as the asymmetry is not too severe. That is, as long as

$$0.276 \approx \frac{1}{2} \left(1 - \frac{1}{\sqrt{5}}\right) < \frac{m}{n} < \frac{1}{2} \left(1 + \frac{1}{\sqrt{5}}\right) \approx 0.724.$$  (5)

The proposition indicates that asymmetric arrangements of bidders into syndicates can also be pro-competitive, as long as neither syndicate contains more than about $3/4$ of the original bidders. The above bounds are not tight unless $n$ is very large. The fewer the number of firms, the greater the level of asymmetry that still offers competitive improvements. With five or fewer firms, facing any two syndicates is more profitable for the auctioneer than independent bidders. In the appendix, we derive the exact bounds for any $n$.

Asymmetric syndicates show the limits of the suboptimal design effect. While information pooling within a syndicate still substitutes for the lack of competition between syndicated bidders, the precision of a syndicate’s private information determines the revenue rankings. Imagine an extremely asymmetric situation in which a larger, almost precisely informed syndicate ($n - m \to \infty$) bids against a small group ($m$ finite). Since the larger syndicate is perfectly informed about the object’s value (having an arbitrarily large amount of signals), the smaller syndicate will win when it overestimates the true value, and will lose (but set the price) when it underestimates the value. Since the prior distribution over signals is uniform and therefore symmetric, it is easy to see that each situation is equiprobable. This means that half the time the price will be determined by the less informed bidder.\textsuperscript{16} Thus, the price will be less informative than in the auction without syndicates. Our results point to this trade-off in terms of bounds on the level of asymmetry.

The benchmark model indicates that syndication can have pro-competitive effects under auction mechanisms that do not reveal rival bidders’ private information. This result requires none of the traditional portfolio-theory justifications for syndicates. Even if all firms have sufficient capital and risk tolerance to bid on their own, syndicates can still be pro-competitive.

\textsuperscript{16} This discussion does not require symmetry but only a positive probability that the less informed syndicate wins in equilibrium. Facing a perfectly informed competitor, the less-informed syndicate will earn zero profits in equilibrium (Milgrom and Weber, 1982b). However, positive profits are earned by the larger syndicate even as the number of its members becomes arbitrarily large. Conversely, bidding without syndicates will, as $n$ becomes large, extract full surplus.
4 General Model

In our benchmark model, the maximum and minimum signals are sufficient statistics for the underlying value. In particular, no matter how many signals a syndicate possesses, its best guess about the value of the item is the average of its maximum and minimum signal. This property is not unique to this model as many non-uniform distributions of signals will yield identical equilibrium bid functions.\(^{17}\)

Define \(s_{(j)}\) as the \(j\)-th highest order statistic of the vector of private signals, \(s\). In this section, we follow Milgrom and Weber (1982a), allowing for a broader class of value functions which depend both on nature’s draw and on the realizations of private information. Consider value functions of the form

\[
v(w, s) = \gamma_0 w + \sum_{i=1}^{n} \gamma_i s_{(i)}
\]

(6)

where \(\gamma_j\) are positive constants. Our benchmark model can be obtained by setting \(\gamma_0 = 1\) and \(\gamma_i = 0, i \geq 1\). As in our benchmark model, an English auction outperforms a second-price auction in this context without syndication. Since the two bidders with the highest signals would observe the drop-out points of bidders with lower signals, much of the uncertainty in the value function would be resolved and competed away between them. In this section, we concentrate on the sealed-bid second-price auction.

Not all parameterizations of this model are analytically tractable in the syndicated case, with both existence and uniqueness of equilibria in question (Mares and Harstad, 2007). It still lends considerable insight into the generality of our results for the benchmark model. For the case without syndicates, we demonstrate in the next claim that the general value functions are bid- and revenue-equivalent to a simple model where

\[
v(w, s) = \alpha s_{(1)} + (1 - \alpha)s_{(n)}
\]

(7)

for an appropriately chosen \(\alpha\). When \(\alpha = 1/2\), this formulation is equivalent in bids and revenues to our benchmark model. Thus, even though the value function is different from our benchmark model (no \(w\) appears on the right hand side of Eq. 7), we obtain identical equilibria.

Claim 4. In a second-price auction without syndicates, any linear (convex) combination of \(w\) and the order statistics of bidders’ signals is equivalent in equilibrium bids and revenue to some linear (convex) function of only \(s_{(1)}\) and \(s_{(n)}\).

\(^{17}\)This is the case in second-price auctions as long as the support of \(s_{|w}\) remains \([w - \theta, w + \theta]\) and \(f(x|w) = h(x - w + \theta / \theta)\) where \(h(\cdot)\) is a power function.
Working with the simplified form of the model allows for closed form solutions in the presence of syndicated bidders. We exploit this property to characterize the revenue effects of syndicated bidding for different values of $\alpha$.

**Proposition 3.** Consider the value function in Eq. (7) and $n \geq 4$. In a second-price auction, there exists an $\alpha^* > 0$ such that the expected revenue from two symmetric syndicates is higher than the expected revenue in the absence of syndicates if and only if $\alpha < \alpha^*$. Furthermore,

1. $\alpha^*$ is decreasing in $n$,
2. $0.618 \approx \frac{1}{2} (\sqrt{5} - 1) \leq \alpha^* \leq \frac{1}{4} (\sqrt{33} - 3) \approx 0.686$, and
3. When $\frac{1}{2} (\sqrt{5} - 1) < \alpha < \frac{1}{4} (\sqrt{33} - 3)$, two symmetric syndicates lead to higher revenues than independent bidders if and only if $n < \frac{2(1-\alpha)}{\alpha^2+\alpha-1}$.

This result is consistent with Proposition 1, but allows us to explore the tradeoffs between the information pooling, competition, and suboptimal design effects more directly. Consider the extreme case where $\alpha = 1$ so the value is equal to the highest signal received by any bidder. The equilibrium bid of a syndicate is simply the maximum signal held by that syndicate. The competition effect is clearly the dominant force in this example since any concentration of information carries with it the chance that the two highest signals are in the same syndicate. In the absence of syndicates, the second-highest signal is the price. With syndicates, there is some probability that even lower order statistics will form the price. Syndicates will therefore lead to lower prices.

A different revenue dynamic is at work when $\alpha = 0$, so the value is equal to the minimum of the signals. The optimal mechanism in this case is actually relatively simple, involving a sealed bid auction where the highest bidder wins the asset and pays the lowest submitted bid (Mares and Harstad, 2007). This mechanism is ex-post incentive compatible and extracts full surplus. The second-price auction generally performs poorly in these circumstances (Mezzetti and Tsetlin, 2009), but when exactly two syndicates compete, the second-price auction becomes equivalent to the lowest-bid auction. The winner of the auction with two syndicates pays the minimum signal of the other syndicate, which is precisely the object’s value. The second-price auction extracts full surplus. Thus, a second-price auction becomes increasingly closer to the optimum with fewer bidders, and extracts full surplus when only two syndicates exist. Facing a highly concentrated information profile with precisely two syndicates, the auctioneer captures greater revenue than if any number of bidders bid independently.
While the maximum-value auction \((\alpha = 1)\) displays the negative force of the competition effect, the minimum value auction \((\alpha = 0)\) highlights the positive, pro-syndicate aspects of the suboptimal design effect. Proposition 3 demonstrates that the suboptimal design effect dominates the competition effect for a majority of the parameter space.

5 Multi-unit auctions

So far, we have assumed that there is one item for auction. In many settings, including underwriting and treasury auctions, bidders request both a price and a quantity. The mechanisms for allocating equity issues range from formal uniform-price auctions to less structured auction-like mechanisms (Hauser, Yaari, Tanchuma, and Baker, 2006). In the context of IPO book-building, Parlour and Rajan (2005) examine when a monopolist seller would introduce rationing, and ask if selling only a fraction of supply can increase total revenues. The answer is a qualified yes, providing some explanation for underwriters rationing in oversubscribed IPO markets. The model they employ is equivalent to our benchmark model, where \(v(w, s_1, \ldots, s_n) = w\), and \(n\) bidders with unit-demands compete in a uniform price auction for \(k\) units of an identical asset.

Multi-unit auctions present additional analytical challenges in mechanism design. First, while we maintain the analytically tractable assumption that each firm initially has unit demands (Bikhchandani and Huang, 1989, Parlour and Rajan, 2005), a syndicate will demand multiple units as it represents multiple firms. Second, one needs to resolve how an English and a second-price auction generalize to multiple units. For an English auction, the natural extension is an ascending auction which ends when the number of remaining active bidders is equal to the supply. For a second-price auction, the sealed-bid uniform-price auction is a poor extension as it is inefficient and leads to demand reduction (Ausubel and Cramton, 2002), with bidders placing a lower bid on each additional as those bids may end up being the price. The Vickrey mechanism, on the other hand, maintains all of a second-price auction’s desirable properties such as efficiency and equilibrium bidding in dominant strategies. In our context, the Vickrey mechanism implies only that a syndicate will place an identical bid for each unit, and one syndicate’s price is determined by the other syndicate’s bid.
Proposition 4. Consider the benchmark model with $n \geq 4$ and an auctioneer selling $k \leq n/2$ identical assets.

1. In a multi-unit ascending (English) auction, an auction with two symmetric syndicates leads to lower expected revenue than an auction without syndicates.

2. There exists a $r^*(n), \frac{1}{4} \leq r^*(n) \leq \frac{5}{12}$, such that, in a multi-unit sealed-bid (Vickrey) auction, an auction with two symmetric syndicates leads to higher expected revenue than an auction without syndicates if and only if $k/n < r^*(n)$.

As in the single-unit case, syndication is never desirable when an open, ascending auction is used. Since bidders already condition on the points at which bidders with lower signals drop out, the price incorporates more than one bidder’s private information. Syndication does not produce sufficient advantages from information pooling to offset the loss of competition.

In the sealed bid case, the price reflects only the information of a single bidder. When bidders sufficiently outnumber the number of assets for sale, we recover our earlier results that syndication raises revenue. This suggests that our results on the competitive effect of syndicates are not unique to single-unit settings. However, the multi-unit case requires some qualification, as syndicates lead to lower revenue when the number of assets is large. With one asset to sell, a second-price auction is used; the price reflects the information of the second-highest bidder, and thus reflects the second-highest signal. With $k$ assets, the $k+1^{th}$ bidder effectively sets the price. As $k$ approaches $n/2$, the signal of the $k+1^{th}$ bidder approaches the median signal which is an unbiased estimate of the object’s true value. Thus, even though a single signal is reflected in the price, that signal becomes closer to the object’s true value. In this case, syndication again offers only modest information pooling effects but causes a loss of competition.

As in the single-unit case, symmetry of syndicate sizes is not required for positive effects of syndicates to exist, as asymmetric cases may also lead to higher prices under syndication. In the multi-unit case, there are two bounds on the amount of asymmetry that may be permitted. First, as in the one-unit case, too much asymmetry can reverse the result, making syndication anticompetitive in sealed bid auctions. Second, we require that the number of objects is not greater than the membership of the smaller syndicate. Overall, syndicated bidding in multi-unit environments (e.g., equity offerings or treasury securities) has similar competitive effects to the single-unit case (e.g., take-overs).
6 Discussion

Past results suggest that when bidders have independently distributed signals, the loss of competition inherent in syndicate bidding unambiguously reduces revenues. The theoretical distinctions between private and common values, different auction mechanisms, and the use of reserve prices, do not alter the anticompetitive effects of syndication (Waehrer and Perry, 2003, Mares and Shor, 2008, 2009). If we further assume symmetry among bidders, revenue equivalence implies that all standard auction formats perform satisfactorily and equally well.

However, in the book-building process of IPOs, auctions of treasury securities, bidding for takeover targets, and other common-value financial markets, bidders are likely to have correlated signals, rendering common auction formats suboptimal. Meanwhile, optimal mechanisms are likely to be quite complicated, and their implementation often challenging or impossible (Crémer and McLean, 1988, McAfee and Reny, 1992). In practice, we observe simple variations on standard auctions for the allocation of financial instruments. Thus, our model restricts the seller to standard auctions. Nevertheless, as a syndicate possesses a multidimensional signal that represents the information of its members, equilibria of the resulting game may fail to exist (Jackson, 2009). We analyze a class of models that are both solvable and sufficiently robust to offer insights into the basic competitive constraints that syndicates face. We show that syndicates can indeed have pro-competitive effects when private information is correlated.

The effects of syndication depend on both the auction mechanism and the specific value function. Pro-competitive benefits of syndication are limited to sealed-bid mechanisms and are absent in open auctions. The intuition behind this mechanism-specific result has to do with the way information revealed in equilibrium is incorporated into the price. In open auctions, bidders update their beliefs about the asset’s value each time another bidder drops out. This information exchange reduces bidders’ uncertainty and increases the resulting price. In sealed-bid auctions that do not allow bidders to update their estimates, syndication partially substitutes for the information loss by allowing groups of bidders to arrive collectively at better estimates. This comes at the cost of reducing the number of active bidders, exerting downward pressure on the price.

What is important for determining the effects of syndication is not whether an auction theoretically conforms to an English or sealed-bid auction, but how much information is revealed during the bidding process. Harstad and Rothkopf (2000) note that many real-world open auctions provide little observable exit and are better approximated by sealed-bid rather than English auction models. Our analysis suggests that the pro-competitive effects of syndication would carry over to settings where exit is unobservable or where the current highest bid is not revealed.
Our results highlight a surprising link between information and competition. When an auctioneer cannot tailor a mechanism to the specific market environment, syndication can sometimes tailor the market environment to the mechanism. We term this phenomenon the *suboptimal design effect*. When signals are upwardly biased or overly optimistic, the smallest signal is a reasonable representation of true value. In these cases, sealed-bid auction formats perform quite poorly in large markets but can be full-surplus extracting (and thus optimal) when only two syndicates exist. In the opposite case when signals are overly pessimistic, the pro-competitive effect vanishes completely. Significant asymmetry in the sizes of syndicates can also cancel out any pro-competitive effect, though only in fairly large markets. Markets with few bidders can benefit even from highly asymmetric syndication.

This analysis suggests a screen for understanding the likely revenue effects of syndication in sealed-bid environments predicated on a thought experiment. Imagine that a bidder receives several estimates of an asset’s value from his analysts. We write these signals on index cards and place them before the bidder in order from lowest to highest but face down, so that their exact values are not yet known. We ask the bidder to commit to a bidding function which depends on the order of these cards, but not their values. For example, a bidder who believes his analysts to be overly optimistic, often overstating the true value, may choose to bid based only on the smallest estimate. In another context, a bidder may elect to concentrate only on the highest estimates, or perhaps the average of all of them. This choice will, of course, depend on the probability distribution of signals and the value, as well as the value function itself. Our result suggests that syndicates are likely to be pro-competitive whenever sufficient weight is put on the lower values. Specifically, if the bidder has only two estimates available, syndicates can be pro-competitive as long as at least about $\frac{1}{3}$ of the weight is placed on the smaller of the two estimates, with no more than $\frac{2}{3}$ going to the higher estimate.

The policy implications of these results should be understood in a wider context. Our paper focuses on the equilibrium effects of syndication on price. Where syndicates also serve a portfolio diversification or capitalization role, for example, this would make them more competitive (or less anticompetitive). Conversely, syndication may increase systemic financial risk and facilitate coordination and collusion among bidders (Klemperer, 2002, Cai et al., 2011). These would diminish from theoretical competitive gains. Additionally, we focus on the case of only two syndicates due to possible non-existence of equilibria in auctions where bidders have multiple signals (Jackson, 2009). While we expect the tradeoffs between

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18 The idea that market concentration facilitates collusion need not translate to syndicates. Even in a market with only two syndicates, collusion would require coordination among and consent of all constituent member firms. Recent empirical analysis finds no link between syndication and the likelihood of collusion (Boone and Mulherin, 2011).
the competition, information pooling, and suboptimal design effects to persist in markets with three or more syndicates, it is certainly possible that the competitive effects are not monotonic in the degree of syndication.

While our focus is on the effects of syndication, we comment briefly on the incentives to form syndicates in light of our results. In private value auctions, both the incentives to syndicate and the mechanisms by which syndicates divide surplus are well understood in standard models (e.g., Mailath and Zemsky, 1991, Waehrer and Perry, 2003). In common value auctions, models of syndicate formation must confront several technical challenges. Generally, syndicates among bidders with similar signals benefit from reducing competition while syndicates among bidders with very different signals benefit from improved information about the target’s value. However, identifying mechanisms for truthful information sharing within syndicates creates extremely demanding incentive requirements (Myerson and Satterthwaite, 1983, Brusco, Lopomo, Robinson, and Viswanathan, 2007). Further, the value of additional information itself varies across different mechanism types (Persico, 2000). Additionally, as ascending bid auctions exhibit a continuum of equilibria (Bikhchandani and Riley, 1991), the willingness of a syndicate to include an outside bidder will depend on which equilibrium is assumed to exist in the absence of such inclusion.

Clearly, when syndication is anticompetitive, there exist some incentives for syndication as the auctioneer’s lower profit is accompanied by higher overall profit for the bidders. However, incentives to form syndicates may exist even when the ultimate result of syndication is pro-competitive. This is because the greater profits obtained by a syndicate may be offset by the losses to outside firms. For example, if the first syndicate formed becomes substantially better informed, it can benefit by capturing most of the industry profit, even if the profit has declined due to its syndication. Next, non-syndicate firms have the most to gain by syndication as they can restore parity and capture half of the (now even smaller) industry profit. Effectively, the sequential path of syndication induces a prisoner’s dilemma by which syndication is profitable at each stage. While questions of syndicate formation are of great theoretical and practical importance, we leave them for future research.

Several antitrust cases have recently challenged the way we analyze the competitive effects of American syndicates. Application of traditional antitrust paradigms to syndication could thrust financial structures into an unfamiliar regulatory environment. Firms may be subject to arguments about theoretical missed gains, criminal statutes governing joint bidding, and heightened civil penalties as federal antitrust suits allow plaintiffs to recover threefold their damages. Yet, all of these rest on the assumption that joint bidding, absent capitalization or risk-sharing needs, skews auction prices in a socially undesirable direction. Our analysis suggests that this need not be the case.
Appendix: Proofs

Lemma 1. The distribution of $w$ conditional on having signal $s$ and on the highest signal among the remaining $n-1$ signals, $y_1$, being equal to $s$ is given by

$$f(w|s, y_1 = s) = \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}}, \quad w \in [s-\theta, s+\theta]. \quad (A.1)$$

Proof. First, note that

$$Pr\{y_1 = s|w, s\} = \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}} \quad (A.2)$$

Then, whenever $|w-s| \leq \theta$, we have by Bayes’ Rule,

$$Pr\{W = w|s, y_1 = s\} = \frac{Pr\{y_1 = s|W = w, s\}Pr\{W = w|s\}}{\int_z Pr\{y_1 = s|W = z, s\}Pr\{W = z|s\}dz} \quad (A.3)$$

$$= \frac{(n-1)(s-w+\theta)^{n-2}}{\int_{s-\theta}^{s+\theta} (n-1)(s-z+\theta)^{n-2}dz} \quad (A.4)$$

$$= \frac{(n-1)(s-w+\theta)^{n-2}}{(2\theta)^{n-1}} \quad \Box$$

We will also make repeated use of the following identities:

$$\int_{s-\theta}^{s+\theta} f(w|s, y_1 = s) \, dw = 1 \quad (A.5)$$

$$\int_{s-\theta}^{s+\theta} w f(w|s, y_1 = s) \, dw = s - \left(\frac{n-2}{n}\right)\theta \quad (A.6)$$

Claim 1. In a second-price auction, the equilibrium bid in the benchmark model without syndicates is given by

$$b(s) = s - \left(\frac{n-2}{n}\right)\theta. \quad (A.7)$$

Proof of Claim 1. We prove a more general result than in the text, that the given bidding function is the equilibrium for any value function given by:

$$v(w, s_1, \ldots, s_n) = \beta w + (1-\beta) \left(\frac{s_1 + s_n}{2}\right) \quad (A.8)$$

where $\beta \in [0, 1]$. When $\beta = 1$ this is the benchmark model, and when $\beta = 0$, this is the average of the maximum and minimum signals. We show that the equilibrium
bid does not depend on $\beta$. Denote by $y_{n-1}$ the lowest signal among the remaining $n-1$ signals.

\[
E[v|s, y_1 = s, w] = \beta w + (1 - \beta) \frac{1}{2} (s + E[y_{n-1}|y_1 = s, w]) 
\]
\[= \beta w + \frac{1 - \beta}{2(n-1)} (ns + (n-2)(w - \theta)) \tag{A.9}
\]
\[= \left(1 - \frac{n(1 - \beta)}{2(n-1)}\right) w + \left(\frac{n(1 - \beta)}{2(n-1)}\right) \left(s - \left(\frac{n-2}{n}\right) \theta\right) \tag{A.10}
\]
\[= w - (1 - \beta) \theta + \left(\frac{n(1 - \beta)}{2(n-1)}\right) (s - w + \theta) \tag{A.11}
\]

Using Eqs. (A.5) and (A.6), and Lemma 1, the equilibrium bid is given by

\[
b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_1 = s)E[v|s, y_1 = s, w] \, dw \tag{A.13}
\]
\[= s - \left(\frac{n-2}{n}\right) \theta - (1 - \beta) \theta + \int_{s-\theta}^{s+\theta} \frac{n(s-w+\theta)^{n-1}}{(2\theta)^n} (1 - \beta) \theta \, dw \tag{A.14}
\]
\[= s - \left(\frac{n-2}{n}\right) \theta \quad \Box
\]

**Claim 2.** In an English auction, the equilibrium drop-out point in the benchmark model without syndicates is given by

\[
b(s) = \begin{cases} 
  s & \text{if no one has yet dropped out} \\
  \frac{1}{2}s + \frac{1}{2}s_{\min} & \text{otherwise} 
\end{cases} \tag{A.15}
\]

where $s_{\min}$ is the price at which the first bidder dropped out.

**Proof of Claim 2.** If $n = 2$, this is equivalent to the second-price auction. For $n \geq 3$, consider bidder 1 with signal $s_1$ and suppose other bidders follow the indicated strategy. Let $s_{\min}$ and $s_{\max}$ be the largest and smallest signals of the other bidders. If $s_1 > s_{\max}$, bidder 1 wins by following the equilibrium strategy and receives a profit of $E[v|s_1, s_{\min}] - b(s_{\max}) = \frac{1}{2} (s_1 - s_{\max}) > 0$. All bids that allow bidder 1 to win result in the same profit. If $s_1 < s_{\max}$, bidder 1 earns zero expected profits at any bid. Alternately, define $y_i$ as the $i^{th}$ highest signal of bidder 1’s $n-1$ rivals.
\( b(s|y_{n-1} = s_{\text{min}}) = \int_w E[v|w, s, y_1 = s, y_{n-1} = s_{\text{min}}] f(w|s, y_1 = s, y_{n-1} = s_{\text{min}}) \, dw \)  

(A.16)

\[
= \int_{s_{\text{min}} + \theta}^{s_{\text{min}} + \theta} \frac{w}{2\theta - (s - s_{\text{min}})} \, dw 
\]

(A.17)

\[
= \frac{(s_{\text{min}} + \theta)^2 - (s - \theta)^2}{4\theta - 2(s - s_{\text{min}})} 
\]

(A.18)

\[
= \frac{s + s_{\text{min}}}{2} 
\]

Claim 3’ (Generalization of Claim 3). With two syndicates, if the value function depends only on \( w, s_1, \) and \( s_n \), then the equilibrium bid is the expected value given one’s signals. In particular, if one syndicate has signals \( \{s_1, \ldots, s_m\}, m \geq 1 \), the equilibrium bid is:

\[
\frac{1}{2} \min\{s_1, \ldots, s_m\} + \frac{1}{2} \max\{s_1, \ldots, s_m\} 
\]

(A.19)

for the benchmark case, and

\[
\alpha \max\{s_1, \ldots, s_m\} + (1 - \alpha) \min\{s_1, \ldots, s_m\} 
\]

(A.20)

for the general model.

Proof. Imagine a syndicate’s rival follows this strategy. There are four possibilities (parts in parentheses refer to the benchmark case):

1. The rival has a higher min and higher max. The object is (expected to be) worth less than the rival’s bid; any bid that assures losing is a best reply.
2. Rival has a lower max and a lower min. The object is (expected to be) worth more than the rival’s bid; any bid that assures winning is a best reply.
3. Rival has a lower max and higher min. Whenever the rival’s bid is lower than one’s own weighted max and min, the bidder wishes to win, but not otherwise.
4. Rival has a higher max and lower min. The rival is bidding the (expected value of the) object’s value. All bids are a best response.

The proposed bidding strategy satisfies all four conditions and, because of item 3, the proposed bidding strategy is a unique best response to a rival following the bidding strategy.
Proposition 1.

1. In an English auction, the expected revenue from two symmetric syndicates is lower than the expected revenue in the absence of syndicates.
2. In a second-price auction, the expected revenue from two symmetric syndicates is higher than the expected revenue in the absence of syndicates.

Proposition 2. Consider $n \geq 3$. A second-price auction with two asymmetric syndicates (of sizes $m$ and $n - m$) leads to higher expected revenue than a second-price auction without syndicates as long as the asymmetry is not too severe. That is, as long as

$$0.276 \approx \frac{1}{2} \left(1 - \frac{1}{\sqrt{5}}\right) < \frac{m}{n} < \frac{1}{2} \left(1 + \frac{1}{\sqrt{5}}\right) \approx 0.724. \quad (A.21)$$

Proof of Propositions 1 and 2.

SYNDICATE REVENUES: Consider a bidder with $m$ signals. We define

$$\bar{s}_m = \frac{1}{2} \min\{s_1, \ldots, s_m\} + \frac{1}{2} \max\{s_1, \ldots, s_m\} \quad (A.22)$$

as the average of the maximum and minimum of the bidder’s $m$ signals. By Claim 3, with two syndicates, a syndicate with $m$ signals bids $s_m$.

We want to calculate $f_{s_m}(s|w)$, the distribution of $\bar{s}_m$ conditional on value. For $m = 1$, this is simply uniform on $[v - \theta, v + \theta]$. Suppose $m \geq 2$.

$$f_{\bar{s}_m}(s|w) = \int_{w-\theta}^{w+\theta} m(m-1)f(x)f(2s-x)|F(x) - F(2s-x)|^{m-2}dx \quad (A.23)$$

where the integration is over $x$, the largest of the $m$ signals. Given the uniform distribution,

$$= \int_{\min\{w+\theta,2s-w+\theta\}}^{\max\{w-\theta,2s-w-\theta\}} \left(\frac{1}{4\theta^m}\right) m(m-1)|s-x|^{m-2}dx \quad (A.24)$$

$$= \begin{cases} \int_{w-\theta}^{s} \frac{m(m-1)(s-x)^{m-2}}{2\theta^m}dx & s \leq w \\ \int_{2s-w-\theta}^{s} \frac{m(m-1)(s-x)^{m-2}}{2\theta^m}dx & s > w \end{cases} \quad (A.25)$$

$$= \begin{cases} -\frac{m(s-x)^{m-1}}{2\theta^m} & s \leq w \\ -\frac{m(s-x)^{m-1}}{2\theta^m} & s > w \end{cases} \quad (A.26)$$

$$= \begin{cases} \frac{m(s-w+\theta)^{m-1}}{2\theta^m} & s \leq w \\ \frac{m(w-\theta-s)^{m-1}}{2\theta^m} & s > w \end{cases} \quad (A.27)$$
And the CDF is given by

\[
F_{\tilde{S}_m}(s|w) = \begin{cases} 
\frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^m & s \leq w \\
1 - \frac{1}{2} \left( \frac{w+\theta-s}{\theta} \right)^m & s > w
\end{cases} \quad (A.28)
\]

Let one syndicate have \(m\) signals, and the other have \(n - m\) signals, with \(1 \leq m \leq n - 1\). Revenue is equal to the expectation of \(\min\{\tilde{s}_m, \tilde{s}_{n-m}\}\), given by

\[
R^{2,m} = \int_{w-\theta}^{w} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} \left( 1 - \frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^{n-m} \right) ds + \int_{w}^{w+\theta} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} \left( \frac{w+\theta-S}{\theta} \right)^{n-m} ds \quad (A.29)
\]

\[
+ \int_{w-\theta}^{w} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} \left( 1 - \frac{1}{2} \left( \frac{s-w+\theta}{\theta} \right)^m \right) ds + \int_{w}^{w+\theta} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} \left( \frac{w+\theta-S}{\theta} \right)^m ds \quad (A.30)
\]

\[
= \int_{w-\theta}^{w} s \frac{m(s-w+\theta)^{m-1}}{2\theta^m} ds + \int_{w-\theta}^{w} s \frac{(n-m)(s-w+\theta)^{n-m-1}}{2\theta^{n-m}} ds \quad (A.33)
\]

\[
- \int_{w-\theta}^{w} s \frac{n(s-w+\theta)^{n-1}}{4\theta^n} ds + \int_{w}^{w+\theta} s \frac{n(s-w+\theta)^{n-1}}{4\theta^n} ds \quad (A.34)
\]

\[
= w - \frac{\theta(n+2)}{2(m+1)(n-m+1)} + \frac{\theta}{2(n+1)} \quad (A.35)
\]

\[
= w - \theta \left( \frac{(n+1)^2 - m(n-m)}{2(m+1)(n-m+1)(n+1)} \right) \quad (A.36)
\]

**NO SYNDICATES REVENUES (second-price):** From Claim 1, the equilibrium bid is

\[b(s) = s - \left( \frac{n-2}{n} \right) \theta \]

Revenue is the above evaluated at the expectation of the second-highest signal:

\[
R^{n,1} = \frac{n(n-1)}{(2\theta)^n} \int_{w-\theta}^{w+\theta} s(w+\theta-s)(s-w+\theta)^{n-2} ds - \left( \frac{n-2}{n} \right) \theta \quad (A.37)
\]

\[
= w - \left( \frac{2(n-1)}{n(n+1)} \right) \theta \quad (A.38)
\]
EFFECT OF SYNDICATES (second-price): Syndicates raise revenue if $R^2,m > R^{n,1}$:

\[
\begin{align*}
\frac{n(n-1)^{2-m(n-m)}}{2(m+1)(n-m+1)(n+1)} \theta > w - \left( \frac{2(n-1)}{n(n+1)} \right) \theta \\
4(m+1)(n-m+1)(n-1) > n(n+1)^2 - mn(n-m) \\
m(n-m)(5n-4) > (n+1)(n^2-3n+4)
\end{align*}
\] (A.39) (A.40) (A.41)

Which is equivalent to

\[
\frac{1}{2} (1 - \phi) < \frac{m}{n} < \frac{1}{2} (1 + \phi)
\] (A.42)

where

\[
\phi = \sqrt{\frac{(n+4)(n^2-4)}{n^2(5n-4)}}
\] (A.43)

Because $\phi$ is positive for $n \geq 2$, we obtain part 2 of Proposition 1. Differentiation reveals that $\phi$ is decreasing with $n$. Taking the limit of $\phi$ as $n \to \infty$ gives the bounds in Proposition 2.

NO SYNDICATES REVENUES (English): From the equilibrium bid in claim 2, revenue is the expectation of the average of the lowest and second-highest signal:

\[
R^{n,1} = \frac{n(n-1)(n-2)}{(2\theta)^n} \int_{w}^{w+\theta} \int_{s}^{w+\theta} \frac{1}{2} (s + s')(w + \theta - s')(s' - s)^{n-3} ds' ds
\] (A.44)

\[
= w - \frac{1}{n+1} \theta
\] (A.45)

EFFECT OF SYNDICATES (English): For part 1 of Proposition 1, we must show that $R^{2,n/2} < R^{n,1}$:

\[
w - \left( \frac{3n+2}{2(n+2)(n+1)} \right) \theta < w - \frac{1}{n+1} \theta
\] (A.46)

which is equivalent to $n > 2$.

The proof of Claim 4 requires the following Lemma.
Lemma 2. Let the value function be given by

\[ v(w, s_1, \ldots, s_n) = \sum_{i=1}^{n} \gamma_i s^{(i)} \]

Where \( \gamma_i \geq 0 \), \( \sum \gamma_i = 1 \), and \( s^{(i)} \) is the \( i \)th highest signal from \( (s_1, \ldots, s_n) \). Define \( \gamma = \sum j \gamma_j \) as the “average” order statistic. The symmetric equilibrium bidding function without syndicates is:

\[ b(s) = s - \left( \frac{\gamma + \gamma_1 - 2}{n} \right) 2\theta. \]

Proof.

\[ E[s^{(j)}|s, y_1 = s, w] = \begin{cases} s & \text{if } j \leq 2 \\ \frac{n+1-j}{n-1}s + \frac{j-2}{n-1}(w - \theta) & \text{if } j > 2 \end{cases} \] (A.47)

\[ E[v|s, y_1 = s, w] = (\gamma_1 + \gamma_2)s + \sum_{j=3}^{n} \gamma_j \frac{n+1-j}{n-1}s + \sum_{j=3}^{n} \gamma_j \frac{j-2}{n-1}(w - \theta) \] (A.48)

\[ = \left( \frac{n+1 - \gamma - \gamma_1}{n-1} \right)s + \left( \frac{\gamma + \gamma_1 - 2}{n-1} \right)(w - \theta) \] (A.49)

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_1 = s)E[v|s, y_1 = s, w] \, dw \] (A.50)

\[ = \int_{s-\theta}^{s+\theta} f(w|s, y_1 = s) \left( s - \left( \frac{\gamma + \gamma_1 - 2}{n-1} \right)(s - w + \theta) \right) \, dw \] (A.51)

\[ = s - \left( \frac{\gamma + \gamma_1 - 2}{n} \right) 2\theta \]

Claim 4. In a second-price auction without syndicates, any linear (convex) combination of \( w \) and the order statistics of bidders’ signals is equivalent in equilibrium bids and revenue to some linear (convex) function of only \( s^{(1)} \) and \( s^{(n)} \).

Proof of Claim 4. By definition, \( 2 \leq \gamma + \gamma_1 \leq n \). Therefore, the bidding function in Lemma 2 satisfies:

\[ s \geq b(s) \geq s - \left( \frac{n-2}{n} \right) 2\theta \] (A.52)

Consider the following value function:

\[ v(w, s_1, \ldots, s_n) = \alpha s^{(1)} + (1 - \alpha) s^{(n)} \] (A.53)
Proposition 3. Consider the value function in Eq. (7) and \( n \geq 4 \). In a second-price auction, there exists an \( \alpha^* > 0 \) such that the expected revenue from two symmetric syndicates is higher than the expected revenue in the absence of syndicates if and only if \( \alpha < \alpha^* \). Furthermore,

1. \( \alpha^* \) is decreasing in \( n \),
2. \( 0.618 \approx \frac{1}{2} (\sqrt{5} - 1) \leq \alpha^* \leq \frac{1}{4} (\sqrt{33} - 3) \approx 0.686 \), and
3. When \( \frac{1}{2} (\sqrt{5} - 1) < \alpha < \frac{1}{4} (\sqrt{33} - 3) \), two symmetric syndicates lead to higher revenues than independent bidders if and only if \( n < \frac{2(1-\alpha)}{\alpha^* + \alpha - 1} \).

Proof of Proposition 3.

SYNDICATE REVENUES A syndicate with \( m \) signals, \( \{s_1, \ldots, s_m\} \), bids:

\[
\bar{s}_m = \alpha \max\{s_1, \ldots, s_m\} + (1 - \alpha) \min\{s_1, \ldots, s_m\} \quad (A.55)
\]

If \( m = 1 \), \( b(s) = s \). For \( m \geq 1 \), following logic similar to the proof of Propositions 1 and 2,

\[
f_{\bar{s}_m}(s|w) = \int_{w-\theta}^{s} m(m-1)f(x)f\left(\frac{s - (1-\alpha)x}{\alpha}\right) \left(F\left(\frac{s - (1-\alpha)x}{\alpha}\right) - F(x)\right)^{m-2} dx
\]

\[
+ \int_{s}^{w+\theta} m(m-1)f(x)f\left(\frac{s - \alpha x}{1-\alpha}\right) \left(F(x) - F\left(\frac{s - \alpha x}{1-\alpha}\right)\right)^{m-2} dx
\]

\[
(A.56)
\]

\[
= \begin{cases} 
\int_{w-\theta}^{s} m(m-1) \left(\frac{1}{z_0}\right)^m \left(\frac{s-x}{\alpha}\right)^{m-2} dx \\
+ \int_{s}^{1-\alpha(s-\alpha(w+\theta))} m(m-1) \left(\frac{1}{z_0}\right)^m \left(\frac{s-x}{\alpha}\right)^{m-2} dx \\
+ \int_{s}^{w+\theta} m(m-1) \left(\frac{1}{z_0}\right)^m \left(\frac{x-s}{1-\alpha}\right)^{m-2} dx & \text{if } s \leq w + (2\alpha - 1)\theta \\
+ \int_{s}^{w+\theta} m(m-1) \left(\frac{1}{z_0}\right)^m \left(\frac{x-s}{1-\alpha}\right)^{m-2} dx & \text{if } s > w + (2\alpha - 1)\theta
\end{cases}
\]

\[
(A.57)
\]

\[
= \begin{cases} 
m \left(\frac{1}{z_0}\right)^m \left(\frac{s-w+\theta}{\alpha}\right)^{m-1} & \text{if } s \leq w + (2\alpha - 1)\theta \\
m \left(\frac{1}{z_0}\right)^m \left(\frac{w-s+\theta}{1-\alpha}\right)^{m-1} & \text{if } s > w + (2\alpha - 1)\theta
\end{cases}
\]

\[
(A.58)
\]
And the CDF is given by

\[
F_s(m|w) = \begin{cases} 
\alpha \left( \frac{s-w+\theta}{2\alpha\theta} \right)^m, & s \leq w + (2\alpha - 1)\theta \\
1 - (1 - \alpha) \left( \frac{w-s+\theta}{2\theta(1-\alpha)} \right)^m, & s > w + (2\alpha - 1)\theta 
\end{cases}
\]  
(A.59)

With symmetric syndicates, both bidders have \(m = n/2\). Revenue is given by

\[
R_{n/2}^2 = \int_{w-\theta}^{w+\theta} 2sf_{s_{n/2}}(s|w)(1 - F_{s_{n/2}}(s|w))ds  
= \int_{w-\theta}^{w+(2\alpha-1)\theta} s\frac{n}{2\theta} \left( \frac{s-w+\theta}{2\theta\alpha} \right)^{n/2-1} ds 
- \int_{w-\theta}^{w+(2\alpha-1)\theta} s\frac{n}{2\theta} \left( \frac{s-w+\theta}{2\theta\alpha} \right)^{n-1} ds 
+ \int_{w+(2\alpha-1)\theta}^{w+\theta} s(1-\alpha)\frac{n}{2\theta} \left( \frac{w-s+\theta}{2\theta(1-\alpha)} \right)^{n-1} ds 
(A.60)
\]

\[
= w + \left( \frac{n(n-3)-2(1-\alpha)(n+1)(n-2)}{(n+1)(n+2)} \right) \theta 
(A.61)
\]

**NO SYNDICATES REVENUES** (second-price): The equilibrium bid is given by

\[
b(s) = s - \left( \frac{n-2}{n} \right) 2(1-\alpha)\theta 
(A.62)
\]

Revenue is the above evaluated at the expectation of the second-highest signal:

\[
R_{n,1}^n = \frac{n(n-1)}{(2\theta)^n} \int_{w-\theta}^{w+n\theta} s(w+\theta-s)(s-w+\theta)^{n-2} ds - \left( \frac{n-2}{n} \right) 2(1-\alpha)\theta 
(A.63)
\]

\[
= w + \left( \frac{n(n-3)\alpha - (n^2+n-4)(1-\alpha)}{n(n+1)} \right) \theta 
(A.64)
\]

**EFFECT OF SYNDICATES**: Syndicates raise revenue if

\[
R_{2,n/2}^n > R_{n,1}^n 
(A.65)
\]

\[
\equiv \frac{n^2-3n-2}{(n+1)(n+2)} - 2(1-\alpha)(n+1)(n-2) > \frac{n(n-3)\alpha - (n^2+n-4)(1-\alpha)}{n(n+1)} 
(A.66)
\]

\[
\equiv \frac{(n+2)(1-\alpha)}{\alpha^2 + \alpha - 1} > 2(1-\alpha)n^2 
(A.67)
\]

\[
\equiv n < \frac{2(1-\alpha)}{\alpha^2 + \alpha - 1}. 
(A.68)
\]
which implies
\[ \alpha < \alpha^* \equiv \frac{1}{2n} \sqrt{(n+2)(5n+2)} - \frac{(n+2)}{2n} \]  
(A.70)

The derivative of \( \alpha^* \) with respect to \( n \) is:
\[ \frac{-3n - 2 + \sqrt{(n+2)(5n+2)}}{n^2 \sqrt{(n+2)(5n+2)}} \]  
(A.71)

It can be confirmed that the numerator is negative and the denominator is positive whenever \( n > 0 \). Thus, \( \alpha^* \) is decreasing with \( n \). The limit of \( \alpha^* \) as \( n \to \infty \) is \( \frac{1}{2} (\sqrt{5} - 1) \approx 0.618 \). When \( n = 4 \) (the lowest \( n \) so that symmetric mergers are possible), \( a^* = \frac{1}{4} (\sqrt{33} - 3) \approx 0.686 \).

**Proposition 4.** Consider the benchmark model with \( n \geq 4 \) and an auctioneer selling \( k \leq n/2 \) identical assets.

1. In a multi-unit ascending (English) auction, an auction with two symmetric syndicates leads to lower expected revenue than an auction without syndicates.

2. There exists a \( r^*(n) \), \( \frac{1}{4} \leq r^*(n) \leq \frac{5}{12} \), such that, in a multi-unit sealed-bid (Vickrey) auction, an auction with two symmetric syndicates leads to higher expected revenue than an auction without syndicates if and only if \( \frac{k}{n} < r^*(n) \).

**Proof of Proposition 4.**

**NO SYNDICATES REVENUES:** Following steps similar to Lemma 1, we derive the distribution of \( w \) conditional on the \( k^{th} \) highest signal among the remaining \( n - 1 \) signals, \( y_k \), being equal to \( s \).

First, note that
\[ \Pr\{y_k = s|w,s\} = \frac{(n-1)!(s-w+\theta)^{n-k-1}(w-s+\theta)^{k-1}}{(k-1)!(n-k-1)!(2\theta)^{n-1}} \]  
(A.72)

Then, by Bayes’ Rule,
\[ \Pr\{W = w|s,y_k = s\} = \frac{\Pr\{y_k = s|W = w,s\} \Pr\{W = w|s\}}{\int_z \Pr\{y_k = s|W = z,s\} \Pr\{W = z|s\} dz} \]
\[ = \frac{(n-1)!(s-w+\theta)^{n-k-1}(w-s+\theta)^{k-1}}{(k-1)!(n-k-1)!(2\theta)^{n-1}} \]  
\[ = \frac{(n-1)!(s-w+\theta)^{n-k-1}(w-s+\theta)^{k-1}}{(k-1)!(n-k-1)!(2\theta)^{n-1}} \]  
\[ \square \]
Next, as in the proof of Claim 1, the equilibrium bid is given by

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_k = s) E[v|s, y_k = s, w] \, dw \]  

(A.73)

\[ b(s) = \int_{s-\theta}^{s+\theta} f(w|s, y_k = s) \, dw \]  

(A.74)

\[ = s - \left( \frac{n-2k}{n} \right) \theta \]  

(A.75)

Which corresponds to our single-unit equilibrium bid when \( k = 1 \). The resulting revenue from \( n \) bidders, 1 signal each, and \( k \) objects available, is the \( k+1 \)th highest bid times \( k \) total objects sold:

\[ R^{n,1,k} = k \int_{w-\theta}^{w+\theta} s \left( \frac{n!(1-F(s|w))^k F(s|w)^{n-k-1} f(s|w)}{(n-k-1)!k!} \right) ds - k \left( \frac{n-2k}{n} \right) \theta \]  

(A.76)

\[ = k \int_{w-\theta}^{w+\theta} s \left( \frac{n!(w-s+\theta)^k (s-w+\theta)^{n-k-1}}{(n-k-1)!k!(2\theta)^k} \right) ds - k \left( \frac{n-2k}{n} \right) \theta \]  

(A.77)

\[ = kw + \left( \frac{n-2k-1}{n+1} \right) k\theta - \left( \frac{n-2k}{n} \right) k\theta \]  

(A.78)

\[ = kw - \left( \frac{2(n-k)}{n(n+1)} \right) k\theta \]  

(A.79)

**SYNDICATE REVENUES:** Since the losing syndicate’s bids determine the price, each syndicate bids the same amount for every unit, and this bid is identical to the single-unit case. Thus, equilibrium revenues are simply \( k \) times the equilibrium revenues in Eq. (A.36) evaluated, for the symmetric case, at \( m = n/2 \).

\[ R^{2,n/2,k} = kw - \left( \frac{3n+2}{2(n+2)(n+1)} \right) k\theta \]

**EFFECT OF SYNDICATES:** Syndicates raise revenue if

\[ R^{2,n/2,k} > R^{n,1,k} \]  

(A.80)

\[ \equiv \frac{2(n-k)}{n(n+1)} > \frac{3n+2}{2(n+2)(n+1)} \]  

(A.81)

\[ \equiv 4(n-k)(n+2) > (3n+2)n \]  

(A.82)

\[ \equiv \frac{k}{n} < r^*(n) = \frac{(n+6)}{4(n+2)} \]  

(A.83)

By inspection, \( r^*(n) \) is decreasing in \( n \), with \( r^*(4) = \frac{5}{12} \) and \( \lim_{n \to \infty} r^*(n) = \frac{1}{4} \).
References


