Game Theory WRITE YOUR NAME ON THE BACK ONLY

#### Game Theory

ECON 208

##### Midterm Examination

This exam has four questions of which you need to answer three, worth 20 points each. There is also an extra credit section. Note that, if you have time, you may attempt all of the questions. All of them will be graded and the best three will be counted.

Please identify any assumptions you are using in your analysis and show all work for partial credit.

You may use the back of a page if necessary, but clearly indicate if you do so that I will know to look there. Work quickly if you wish to answer every question, but carefully.

Question 1.

Consider the following strategic-form simultaneous game. Player 1’s payoffs are listed first, in bold.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Player 2 | | |
|  |  | X | Y | Z |
| **Player 1** | **A** | **60** , 90 | **80** , 90 | **0** , 80 |
| **B** | **90** , 0 | **50** , 50 | **20** , 80 |
| **C** | **80** , 10 | **90** , 0 | **40** , 40 |

1. [4 pts] Does either player have any strictly dominated strategies? If yes, list it/them.
2. [3 pts] Does either player have any weakly dominated strategies? If yes, list it/them.
3. [8 pts] List all pure strategy Nash equilibria of the above game?
4. [2 pts] Is the following statement true or false in general (explain your answer): A strictly dominated strategy can never be a best response.
5. [2 pts] Define “common knowledge” as it is used in game theory.
6. [1 pts] Briefly define or provide examples of “refinements” of Nash equilibria and their use in game theory.

Question 2.

Consider the following game in which two athletes are considering the use of banned substances.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Athlete B | |
|  |  | Use | Don’t Use |
| Athlete A | Use | 0 , 0 | 12 , -2 |
| Don’t Use | -2 , 12 | 10 , 10 |

1. [4 pts] Formally confirm that this game is a prisoner’s dilemma.
2. [8 pts] Imagine that the athletes expect to play this game each year and forever. What must be true of the interest rate (numerically, what must the range of interest rates be) for the athletes to cooperate using the grim trigger strategy?
3. [3 pts] Given your answer above, is the grim trigger strategy likely to support cooperation? Why or why not?
4. [3 pts] Would you expect tit for tat to be sufficient punishment to maintain cooperation in this game? Explain (calculations are not necessary).
5. [2 pts] The above game is *symmetric*, meaning that the payoffs stay the same if roles are reversed. Realistically, games are not symmetric, as each player might different payoffs in each of the scenarios above. In words, how would you analyze the sustainability of cooperation if players are not symmetric?

Question 3.

In class, we discussed the “penalty kicks game.” When played among professionals, the probability of scoring is always greater than one-half. The payoffs for one kicker and goalie are illustrated below.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Goalie | |
|  |  | Left | Right |
| Kicker | Left | 0.6 , 0.4 | 1.0 , 0.0 |
| Right | 0.9 , 0.1 | 0.7 , 0.3 |

1. [3 pts] Does the game pictured above have a Nash equilibrium?
2. [6 pts] In the game above, what is the *Kicker*’s equilibrium strategy?
3. [6 pts] In the game above, what is the *Goalie*’s expected equilibrium payoff?
4. [5 pts] The 1000 employees of the EconoPlant all live in Quiettown, 20 miles away. There are two roads from Quiettown to the EconoPlant. Wide Road has a low speed limit, but is sufficiently wide so that traffic does not cause a slowdown. The commute on Wide Road takes 40 minutes. Narrow Road has a higher speed limit, but congestion can slow traffic. The commute on Narrow Road takes 20+*m*/10 minutes, where *m* is the number of drivers that take that path. If employees aim to minimize commute time, what is the equilibrium commute time on Narrow Road? Briefly explain.

Question 4.

Consider the following game. Player 1’s payoffs are listed first, in bold:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Player 2 | | |
|  |  | X | Y |
| **Player 1** | **A** | **90** , 1 | **10** , 0 |
| **B** | **10** , 0 | **50** , 1 |
| **C** | **100** , 0 | **80** , 1 |

1. Imagine that player 1 makes a decision first and Player 2 makes a decision after observing player 1’s choice. What is the subgame-perfect equilibrium of this game?
2. Imagine that player 2 makes a decision first and Player 1 makes a decision after observing player 2’s choice. What is the subgame-perfect equilibrium of this game?
3. [2 pts] Are your answers to (a) and (b) above the same or different? Explain.
4. [2 pts] What is the meaning of “simultaneous” in game theory?
5. [2 pts] Briefly explain below what is Arrow’s Impossibility Theorem?