

Spring 2025

Game Theory

ECON 3208

MIDTERM EXAMINATION

SOLUTIONS

IMPORTANT:

These are very brief solutions that do not always include complete explanations and therefore do not necessarily constitute answers that would receive full credit

QUESTION 1.

Consider the following strategic-form simultaneous game. Player 1's payoffs are listed first, in bold.

		Player 2		
		X	Y	Z
Player 1	A	5 , 5	3 , 4	80 , 4
	B	2 , 5	5 , 5	70 , 5
	C	2 , 20	1 , 10	5 , 5

- a. [3 pts] List all **strictly dominant** strategies for both players. Briefly explain your answer.

None (no strategy yields a strictly higher payoff than all other strategies available to that player for all strategies of the other player)

- b. [4 pts] List all **strictly dominated** strategies for both players. Briefly explain your answer.

C (by A)

- c. [3 pts] List all **weakly dominated** strategies for both players. Briefly explain your answer.

C (by A); Y and Z (by X)

- d. [6 pts] List all pure-strategy Nash equilibria of the above game.

(A,X) and (B,Y)

- e. [2 pts] Can a Nash equilibrium involve a player playing a strictly dominated strategy? If yes, provide an example. If no, briefly explain why not.

No - A Nash equilibrium is when all players are playing a best reply and a strictly dominated strategy is never a best reply

- f. [2 pts] The United States senate is currently playing a game with respect to the United States budget. The game might look like the one pictured below.

		Democrats	
		Compromise	Don't compromise
Republicans	Compromise	5 , 5	0 , 30
	Don't compromise	30 , 0	-50 , -50

How would you classify this game (i.e., prisoner's dilemma, game of assurance, etc.)? Explain.

Game of chicken (or brinkmanship) where "Compromise" is like "swerve" and "Don't Compromise" is like "Don't Swerve"

QUESTION 2.

Consider the following game:

		P 2	
		X	Y
P 1	A	5 , 20	10 , 0
	B	20 , 20	0 , 200

- a. [9 pts] In the game above, what is Player 1's Nash equilibrium strategy?

Play A 9/10 of the time, B 1/10 of the time

- b. [3 pts] In equilibrium, what is Player 2's expected payoff from strategy Y?

20 (same as from X)

- c. [2 pts] If the "10" in the top right box increased slightly (but everything else stayed the same), how would Player 1's equilibrium strategy change (Would Player 1 play A more often, less often, or just as often?) Explain. [Calculations are not necessary but count as an explanation]

Player 1's equilibrium strategy wouldn't change (Player 1 would play A just as often).
In a mixed strategy equilibrium, Player 1's strategy depends only on Player 2's payoffs.

- d. [3 pts] Find all values of X and Y for which the game below has at least one Nash equilibrium.

		P 2	
		C	D
P 1	A	5 , 6	8 , 4
	B	10 , 2	x , Y

X and Y can be any value since all games have a Nash equilibrium

- e. [3 pts] An online reading discussed data from penalty kicks in professional soccer. Soccer players tend to kick to the left side of the goal about 40% of the time and to the right side of the goal about 60% of the time. When kicking to the left, soccer players score a goal 80% of the time. How often do they score a goal when kicking to the right? Briefly explain.

80% (same in both directions to be indifferent)

QUESTION 3.

Consider the following game. Player 1's payoffs are listed first, in bold:

		Player 2	
		X	Y
Player 1	A	10 , 6	20 , 2
	B	5 , 8	30 , 8

- a. [2 pts] Write down all pure-strategy Nash equilibrium/equilibria of the above game.

(A,X) and (B,Y)

- b. [10 pts] Imagine that Player 1 makes a decision first and Player 2 makes a decision after observing Player 1's choice. Write down every subgame-perfect Nash equilibrium of this game.

(A;X,X) and (B;X,Y). If Player 1 chooses B, Player 2 is indifferent between X and Y, leading to two (pure-strategy) subgame-perfect Nash equilibria.

- c. [3 pts] Imagine that Player 2 makes a decision first and Player 1 makes a decision after observing Player 2's choice. Write down every subgame-perfect Nash equilibrium of this game.

(Y;A,B)

- d. [2 pts] Of the above three ways of playing this game, how would the players prefer to play it? Briefly explain.

Multiple answers possible, but Player 2 playing first has a unique equilibrium that maximizes both players' payoffs while the other two ways have multiple equilibria, including ones the players don't prefer.

- e. [3 pts] Imagine that player 1 decides on A or B in March and Player 2 decides on X or Y in April. Discuss whether this game should be characterized as simultaneous or sequential.

Depends on information.

If Player 2 knows Player 1's decision, it is sequential. Otherwise, it is simultaneous.

QUESTION 4

- a. [5 pts] What does it mean for payoffs in a game to be “common knowledge”?

Both players know the payoffs; both players know that both players know the payoffs; both players know that both players know that... an infinite number of times.

- b. [5 pts] Discuss how a game theory student should approach very long sequential games (with many rounds or decision points) when playing against those who perhaps haven't studied game theory.

Don't expect the other player to use backward induction until only 2-3 moves remain. Assume that they will play intuitively and not strategically at first, and thus it may be beneficial for the game theory student to take advantage of that early intuitive play (by not playing the equilibrium strategy at first).

- c. [10 pts] The nine-person International Olympic Committee is deciding between three possible host cities for the 2036 Summer Games: Barcelona (B), Copenhagen (C), and Doha (D). The rules of voting are as follows: (1) in the first round, each of the nine members votes for a city, B, C, or D; (2) the city with the fewest votes is eliminated and, in the second round, each of the nine members votes for one of the two remaining cities, with the larger vote-getter winning.

Two committee members prefer C to B to A, and will be voting for C in the first round

Three committee members prefer B to A to C, and will be voting for B in the first round

Four committee members prefer A to C to B

How should the four committee members vote? Carefully explain.

Vote for B or C in the first round:

- If vote for A, C eliminated (4 for A, 3 for B, 2 for C), then A versus B in second round yields B
- If vote for B, A eliminated (0 for A, 7 for B, 2 for C), then B versus C in second round yields C
- If vote for C, A eliminated (0 for A, 3 for B, 6 for C), then B versus C in second round yields C

Voting for A leads to B winning, which is the worst outcome for the four committee members. There is no way to achieve A as the final outcome (since B cannot be eliminated in the first round and B beats A). By voting for either B or C in the first round and eliminating A, the four committee members get their second choice of C when C beats B in the second round.