

Game Theory & Business Strategy

MGT 425

MIDTERM EXAMINATION

Solutions

These are sample solutions. They are not guaranteed to be correct, nor necessarily reflective of the complete answers required to get full credit.

QUESTION 1.

		Mindy		
		X	Y	Z
Mork	A	80 , <u>30</u>	0 , 20	<u>20</u> , 20
	B	85 , 0	<u>20</u> , <u>20</u>	5 , 5
	C	<u>95</u> , <u>25</u>	5 , <u>25</u>	5 , <u>25</u>

- Does either player have a strictly dominant strategy? If yes, list it/them. Briefly explain.
Neither player has a dominant strategy. No strategy receives higher payoffs than all others for any of the opponent's strategies.
- Does either player have a strictly or weakly dominated strategy? If yes, list it/them. Briefly explain.
Neither player has a strictly dominated strategy as no strategy receives a strictly lower payoff than some other strategy for any of the opponent's strategies. However, Z for Mindy is weakly dominated (by Y).
- List all pure strategy Nash equilibria of the above game.
(C,X) and (B,Y). See underlining of best responses in the game above.
- If Mindy's payoffs in the top-left box (A,X) increased from 30 to 300, how would your answer to part c above change? Explain.
It would not change. Mindy's best response to A would remain X. Mork's best response to X would remain C. Thus, (A,X) would not be an equilibrium regardless of what payoff Mindy earns.
- Is the following statement true or false (explain your answer): In any strategic-form game, if a player has a dominant strategy, then all of the other strategies of that player are dominated.
True. This is the definition of a dominant strategy. A strategy is dominant if it dominates all other strategies. Thus, all other strategies are dominated.

QUESTION 2.

You and another firm are engaged in a joint venture. Each year, you and the other firm decide whether to invest effort in the joint venture (which is costly), or simply free-ride on the effort of the other firm. The game is represented below.

		Other Firm	
		Effort	No Effort
You	Effort	25 , 25	0 , 50
	No Effort	50 , 0	10 , 10

- a. Formally confirm that this game is a prisoner's dilemma.
1. *Each player has a dominant strategy (No Effort)*
 2. *The resulting equilibrium results in lower payoffs for both players (10) than they could earn if they played their dominated strategies (25)*

- b. Imagine that you expect to play this game with the Other Firm each year and forever. The Other Firm employs a grim trigger strategy, cooperating each year only if you have been cooperative in every previous year. What must be true of the interest rate (numerically, what must the range of interest rates be) for cooperation to be sustainable?

Cooperating in each period earns a payoff of 25.

Cheating earns a payoff of 50 in the first period, but then the other player will play No Effort in perpetuity, so the future payoffs would be 10 per year.

In order to sustain cooperation, we require:

<i>Present value of cooperating</i>	<i>> Present value of cheating</i>
<i>25 in each period</i>	<i>> 50 in the first period, 10 in each future period</i>
$25 + 25/r$	$> 50 + 10/r$
$15/r$	> 25
$15/25$	$> r$
0.6	$> r$

Thus, we will be able to sustain cooperation if $r < 0.6$ (or $r < 60\%$).

- c. In one or two sentences, given your answer in part c., remark on how likely it is that cooperation will be sustained.

Cooperation has a "return" of 60% per year. Firms will cooperate if this is a better investment than alternatives. It is unlikely that firms have alternative investments yielding over 60%, so cooperation is very likely.

- d. Imagine now that, if you play non-cooperatively, the Other Firm will not be able to observe this for two years. That is, the other firm still plays the grim trigger strategy, but does not begin punishment for non-cooperative behavior until two years later. According to the online reading, does this make cooperation more or less likely? Briefly explain.

This implies that cheating is now more profitable (as I can get away with it for two years, instead of one). Thus, cooperation is harder to sustain when cheating is more difficult to monitor.

QUESTION 3.

Criminals escape from the Island of Marou either by the East Bridge or the West Bridge. The Sheriff can monitor only one bridge at a time. Crossing the West Bridge is a more treacherous, dangerous journey (half of those who try to cross die on the way). The Sheriff is paid \$1,000 whenever he captures a criminal, but not when a criminal dies on the way. Each player tries to maximize his payoffs – the chance of escape for the criminal and the expected bonus payment for the Sheriff. The resulting game is represented below.

		Sheriff	
		(q) East	(1-q) West
Criminal	(p) East	0.0 , <u>1000</u>	<u>1.0</u> , 0
	(1-p) West	<u>0.5</u> , 0	0.0 , <u>500</u>

Note that the game does not have a pure strategy equilibrium. Thus, the equilibrium must be in mixed strategies. Denote (as I did above) by p the probability of East for the Criminal, and by q the probability of East for the Sheriff. The criminal's expected payoffs are:

$$\text{East: } 0q+1(1-q) = 1-q \qquad \text{West: } 0.5q+0(1-q) = 0.5q$$

Setting these equal reveals the Sheriff's equilibrium strategy: $1-q = 0.5q \rightarrow q=2/3$

Next, the Sheriff's expected payoffs are:

$$\text{East: } 1000p + 0(1-p) = 1000p \qquad \text{West: } 0p + 500(1-p) = 500-500p$$

Setting these equal reveals the Criminal's equilibrium strategy: $1000p = 500-500p \rightarrow p=1/3$

- a. Write down below the unique equilibrium of the above game.

It is important to recall correctly what "p" and "q" stood for in writing down the equilibrium.

The equilibrium is:

Criminal goes East 1/3 of the time and West 2/3 of the time

Sheriff goes East 2/3 of the time and West 1/3 of the time

- b. What is the criminal's chance of escape in equilibrium?
 c. What is the Sheriff's expected bonus payment in equilibrium?

For b and c, we are being asked for the players' expected equilibrium payoffs. Simply plug your q and p back into the expected payoffs calculated above:

The criminal's expected payoffs are:

$$\text{East: } 1-q = 1/3 \qquad \text{West: } 0.5q = 1/3$$

The Sheriff's expected payoffs are:

$$\text{East: } 1000p = 1000/3 \qquad \text{West: } 500-500p = 1000/3$$

This does two things. First, by getting the same answer for both of each player's strategies, we have confirmed that we calculated p and q correctly. Second, it tells us each player's expected payoffs: (part b.) 1/3, (part c.) \$333.33

- d. To encourage higher effort on the part of the Sheriff, his bonus is doubled. Every time he captures a criminal, he receives \$2,000. That is, the Sheriff's payoffs in the above game are all doubled. How does this change the equilibrium? Explain.

It doesn't. Recall that we find an equilibrium by setting the expected payoffs from each strategy equal to each other. If the payoffs all double, this will cancel. I.e., the Sheriff's payoffs are now:

East: 2000p West: 1000-1000p, but setting these equal to each other allows us to divide both sides by 2 and come to the original equation.

QUESTION 4.

Consider the following game. Player 1's payoffs are listed first, in bold:

		Player 2	
		X	Y
Player 1	A	90 , 1	10 , 0
	B	10 , 0	50 , 1
	C	100 , 0	80 , 1

- a. Imagine that player 1 makes a decision first and Player 2 makes a decision after observing player 1's choice. What is the subgame-perfect equilibrium of this game?

One could draw a game tree which would yield analysis equivalent to the following:

If P1 chooses A P2 chooses X ($1 > 0$) giving player 1 a payoff of 90

If P1 chooses B P2 chooses Y ($0 < 1$) giving player 1 a payoff of 50

If P1 chooses C P2 chooses Y ($0 < 1$) giving player 1 a payoff of 80

Thus, player 1 would choose A ($90 > 80$ and $90 > 50$) so the SPNE is $\{A; X, Y\}$

- b. Imagine that player 2 makes a decision first and Player 1 makes a decision after observing player 2's choice. What is the subgame-perfect equilibrium of this game?

If P2 chooses X P1 chooses C ($100 > 90$, $100 > 10$) giving player 2 a payoff of 0

If P2 chooses Y P1 chooses C ($80 > 50$, $80 > 10$) giving player 2 a payoff of 1

Thus, player 2 would choose Y ($1 > 0$) so the SPNE is $\{Y; C, C\}$

- c. Are the outcomes in (a) and (b) above the same or different? Explain.

They are different. In this game, both players do better going first than going second (player 1 makes 90 instead of 80 and player 2 makes 1 instead of 0).

- d. Briefly explain below what is Arrow's Impossibility Theorem?

See slides for details. A voting procedure does not exist that eliminates the possibility of strategic behavior. Specifically, if you want a voting rule to satisfy two very simple and reasonable properties, only a dictator can achieve that in all cases.