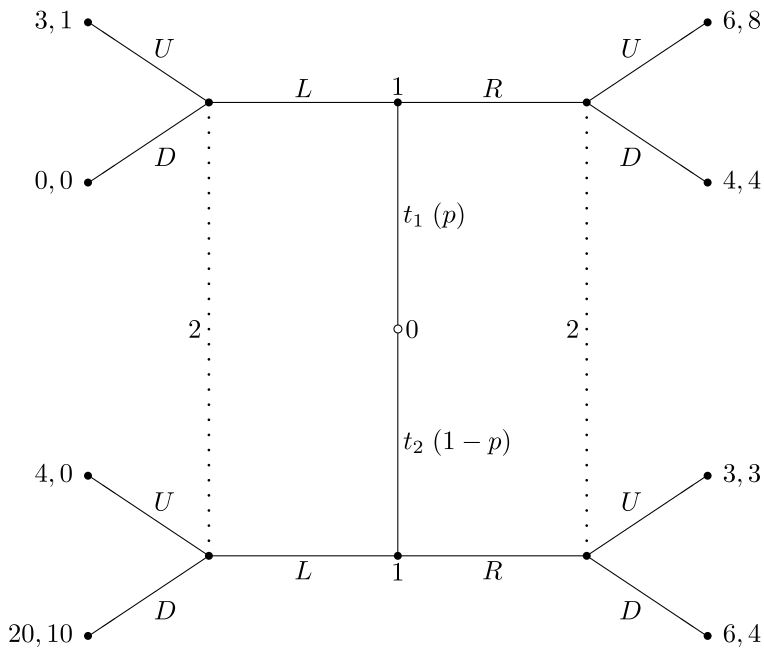
Microeconomic Theory II **Spring 2025**
Final Exam **Mikhael Shor**

Carefully explain and support your answers.

Question 1. Consider the following game. First, nature (player 0) selects t_1 with probability p, $0 < p < 1$, or t_2 with probability $1-p$. Next, player 1 selects L or R. Lastly, player 2 selects U or D.



Find all values of p for which a **pooling** weak Perfect Bayesian equilibrium exists and show one such equilibrium.

Question 2. Consider the Spence signaling model in which a worker is either a high type ($t = H$) or a low type ($t = L$), each with equal probability. The high type has productivity 4 and the low type has productivity 2. The cost of obtaining a level of education, e, is $c_H(e) = \frac{1}{8}e$ for the high type and is $c_L(e) = \frac{1}{4}e$ for the low type. Wage, w, equals a worker's expected productivity. A worker's utility function is $w - c_t(e)$.

A separating equilibrium involves each type, t, choosing a level of education e_t, where $e_L \neq e_H$. What is the range of e_H for which a **separating** weak Perfect Bayesian equilibrium exists?

Question 3. Consider a principal-agent problem in which the agent chooses between two levels of effort, $\{e_l, e_h\}$. The principal pays the agent a wage $w_s \geq 0$ in state s and realizes output of π_s. There are four states, with output levels $(\pi_1, \pi_2, \pi_3, \pi_4) = (0, 3, 9, 18)$. The probability of a state s (or output π_s) conditional on the agent's effort is given by:

	π_1 0	π_2 3	π_3 9	π_4 18
e_l	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{1}{9}$	$\frac{2}{9}$
e_h	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{4}{9}$

The agent's utility is $u(w, e) = \ln w - c(e)$, where $c(e_h) = \ln 4, c(e_l) = \ln 2$, and the agent's reservation utility is 0. The principal is risk neutral with utility in state s given by $\pi_s - w_s$.

(a) Determine the wage schedule that optimally implements e_l and the wage schedule that optimally implements e_h when effort is *observable*.

(b) Determine the wage schedule that optimally implements e_l when effort is *unobservable*.

(c) Determine the wage schedule that optimally implements e_h when effort is *unobservable*.

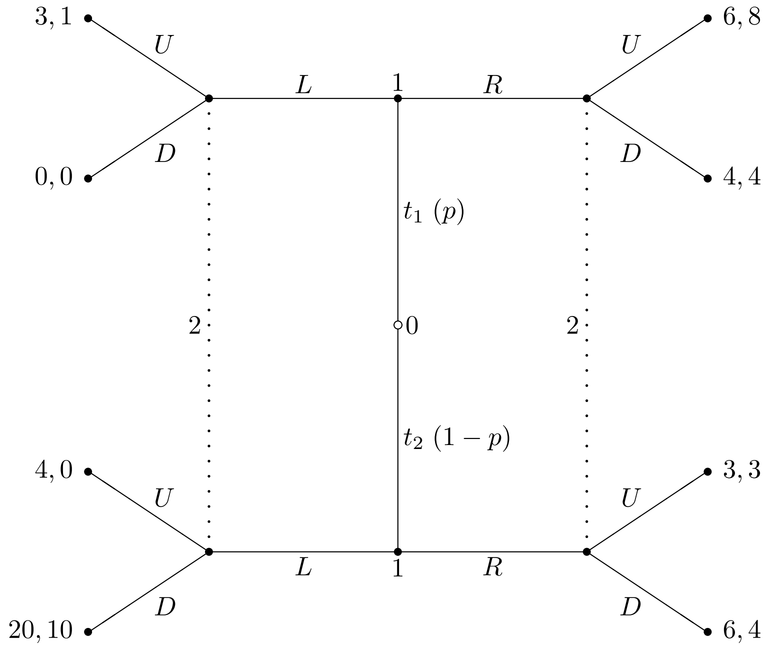
(d) Determine the optimal wage schedule when effort is *unobservable*.

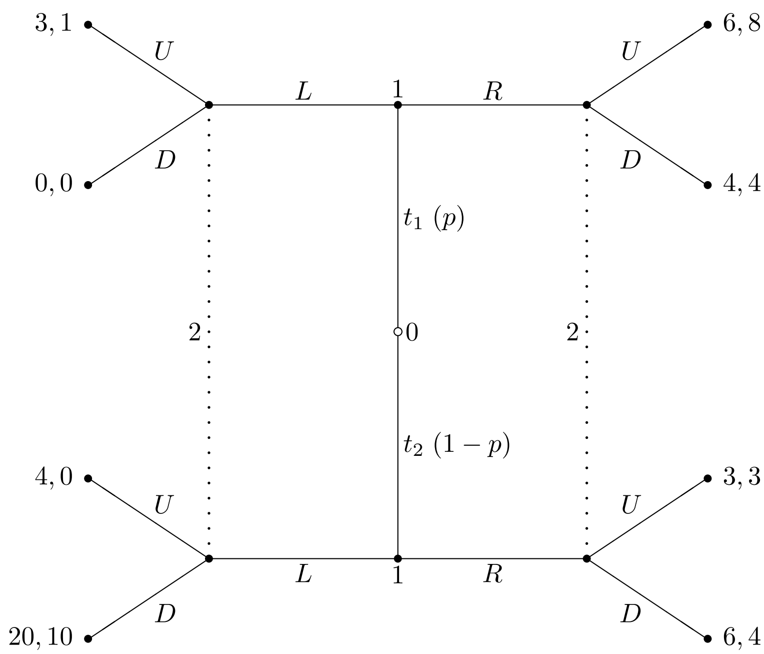
(e) Is the principal's profit higher under observable effort, higher under unobservable effort, or are the profits the same? Briefly explain intuitively.

Question 4. Two drivers are traveling side-by-side in two lanes on a highway that are narrowing due to a merge. As the lanes narrow, the drivers are increasingly likely to crash in the next minute unless one of them yields (slows down) and allows the other driver to pass. Each driver simultaneously selects a time $t_i \in [0, 1]$ to yield if the other driver has not yet yielded.

The probability of a crash is $\min(t_1, t_2)$. The payoff for driver i is -1 if they crash, 1 if they do not crash and driver i is in front ($t_i > t_j$) and 0 if they do not crash and driver i is behind ($t_i < t_j$), with ties resolved randomly.

Find all pure-strategy Nash equilibria.


3, 1
U
L
1
R
U
6, 8
0, 0
D
D
4, 4
t_1 (p)
2
0
2
t_2 ($1-p$)
4, 0
U
U
3, 3
L
1
R
20, 10
D
D
6, 4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