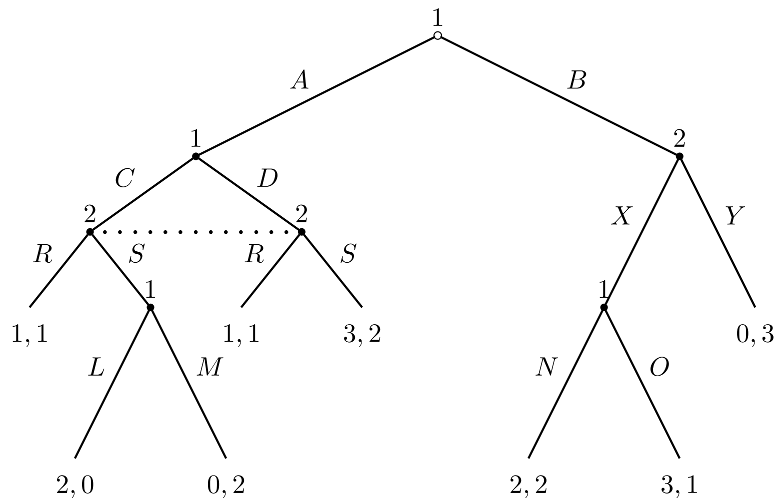
Microeconomic Theory II **Spring 2024**

Midterm Exam **Mikhael Shor**

Solutions
Quick, Incomplete, and not Guaranteed

Question 1. Consider the following extensive form game. The dotted line represents an information set.



(a) Write down a sample pure strategy for each player.

`Player 1 has four information sets with the strategy space given by` $\{A,B\}\times\{C,D\}\times\{L,M\}\times\{N,O\}$ `and Player 2 has two information sets with the strategy space given by` $\{R,S\}\times\{X,Y\}$`. A sample strategy for Player 1 is` A,C,L,N `and for Player 2 is` R,X`.`

(b) List all pure-strategy subgame-perfect Nash equilibria.

`There are two PS SPNE:` $\{A,C,L,O;R,Y\}$ `and` $\{A,D,L,O;S,Y\}$`.`

`On the right side, Player 1 would select O (3>2) and Player 2 would select Y (3>1).`

`On the left side, Player 1 would select M (2>0) which yields the following game:`

		Player 2	
		R	S
Player 1	C	$1,1$	$2,0$
	D	$1,1$	$3,2$

which has two pure-strategy Nash equilibria: $\{C, R\}$ and $\{D, S\}$. Lastly, since for Player 1, playing B yields less than playing A for either of the identified Nash equilibria, both PS SPNE involve player 1 playing A.

(c) Does this game contain a mixed-strategy subgame-perfect Nash equilibrium? Explain why or why not.

Yes, but only involving a mixed strategy for one of the players

Since there is no ‘‘tie" when evaluating any information set, the only way that a mixed strategy SPNE would exist in this game is if there is a mixed strategy NE in this game:

		Player 2: R	Player 2: S
Player 1	C	$1, 1$	$2, 0$
	D	$1, 1$	$3, 2$

Note that D weakly dominated C. No equilibrium can exist in which Player 2 mixes between R and S since Player 1 would respond with D but Player 2 would then be unwilling to mix. For Player 1 to be willing to mix, Player 2 must play R. Then, player 1's best response is any mixture and player 2's best response is R as long as $\sigma_1(C) \geq \frac{1}{2}$. Therefore: $\{A, pC + (1-p)D, L, O; R, Y\}$ is a SPNE for $p \geq 1$.

Question 2. Ten identical firms are deciding whether or not to enter a new industry. Competition occurs over two periods:

- In the first period, firms simultaneously choose whether or not to enter the industry. Entry requires a fixed payment of $F = 196$.
- In the second period, upon observing the number of entrants, entered firms compete a la Cournot by simultaneously selecting quantities, q_i. Profit of firm i is given by $p(Q)q_i$ where $p(Q) = 60 - Q$ and Q is the total quantity of all firms. Firms that do not enter naturally produce nothing and earn a profit of zero.

Determine all subgame-perfect Nash equilibria of this game.

The SPNE are such that any three firms choose $\{Enter, q(n) = \frac{60}{n+1}\}$ and the remaining seven firms choose $\{NotEnter, q(n) = \frac{60}{n+1}\}$ where n is the number of firms that enter in a given subgame.

First, consider what a strategy for a given firm looks like. In the first period, the firm chooses either Enter or Not Enter. In the second period, for every possible pattern of entries (or, by symmetry, for every number of entrants), the firm selects a number q_i.

We solve for period 2 first. Profit of firm i is given by

$$q_i p(Q) = q_i(60 - Q) = q_i(60 - q_i - Q_{-i})$$

Maximizing with respect to q_i yields the best response function

$$q_i = 30 - \frac{1}{2}Q_{-i}.$$

We know that the Cournot equilibrium is unique and symmetric, so

$$q_i^* = 30 - \frac{1}{2}(n-1)q_i^*$$

and therefore

$$q_i^*(n) = \frac{60}{n+1}$$

which is our equilibrium second period strategy. The resulting price is $\frac{60}{n+1}$ and profit is $\left(\frac{60}{n+1}\right)^2$.

In the first period, firms choose between entering or not entering Calculating the profit of entering when $n = 1$ ($30^2 = 900$), $n = 2$ ($20^2 = 400$), $n = 3$ ($15^2 = 225$) and $n = 4$ ($12^2 = 144$) indicates that entry is only profitable (given the 196 cost of entry) if three or fewer firms enter. More directly, entry is profitable if

$$\left(\frac{60}{n+1}\right)^2 > 196 = 14^2$$

or if $60 > 14(n+1)$ or if $n < \frac{23}{7}$ which implies $n \leq 3$. Therefore, a best response in the first period is to enter if no more than two other firms are entering, and to not enter otherwise.

Note 1 : A *best response* is a functions of other players' strategies, but an *equilibrium strategy* in a simultaneous game is *not* a function of other players' strategies. Thus, in the second period, equilibrium quantities are just numbers (which depend on n) and *not* functions of other players' strategies.

Note 2 : When analyzing the first period, firms' profits should not contain q_i or Q which reflect firms' second-period strategies. By substituting second-period strategies, profits should be a function only of whether each firm enters or does not enter.

Note 3 : In the first period, a firm is deciding between entering or not entering. Therefore, best replies should be determined simply by comparing the profit from entry and the profit from not entering.

Question 3. Consider the following normal form game.

		Player 2	
		L	R
Player 1	A	$X, 6$	$16,\ 5$
	B	$20, 6$	$40,\ 5$
	C	$36, 6$	$8, 50$

(a) For what values of X is strategy A strictly dominated? Demonstrate by identifying the necessary conditions for strategy A to be dominated/not dominated.

`A` is strictly dominated if there exists some mixture of B and C that earns strictly higher payoffs for each of Player 2's pure strategies. Thus, A is dominated if there exists a $p \in [0,1]$ such that $20p{+}36(1{-}p) > X$ and $40p{+}8(1{-}p) > 16$. The second condition reduces to $p > \frac{1}{4}$. Evaluating the first at $p = \frac{1}{4}$ (since the left hand side is decreasing in p) yields $X < 32$.

(b) For what values of X is A never a best reply? Demonstrate by identifying the necessary conditions for strategy A to be a best reply/not be a best reply.

A is a best reply to some strategy of Player 2 $pL + (1-p)R$ if $Xp{+}16(1{-}p) \geq 20p{+}40(1{-}p)$ and $Xp{+}16(1{-}p) \geq 36p{+}8(1{-}p)$ which we can rewrite as $p \geq \frac{24}{4+X}$ and $p \leq \frac{8}{44-X}$. Therefore, A is a best response to some strategy if there exists a p satisfying both conditions, which is possible if $\frac{8}{44-X} \geq \frac{24}{4+X}$ or $X \geq 32$. Therefore, A is never a best reply if $X < 32$.

(c) For what values of X is A consistent with rationality? Briefly explain.

$X \geq 32$.

A strategy is consistent with rationality if (depending on the definition one is using) it is undominated or it is a best reply to some beliefs. However, in a two player game, ''never a best reply" and ''strictly dominated" coincide, which is why the above two questions must yield the same answer.

(d) For what values of X is A consistent with both rationality and common knowledge? Carefully explain.

This requires either that the strategy survives the elimination of strictly dominated strategies or the iterated deletion of strategies that are never a best reply. That requires first that A is itself undominated or a best response (same as parts b anc c above). Further, B is clearly undominated. If C is dominated, continued iteration would delete R then B, still leaving A. Therefore, the answer is the same as parts a and b).

(e) For what values of X is A Player 1's unique strategy consistent with rationality and common knowledge? Carefully explain.

This requires that IDSDS (or iterated deletion of strategies that are never a best reply) eliminates B and C. Since B can't be dominated, we need C do be dominated, which requires X>36. Then, R is eliminated, and then B, leaving only A.

(f) ** For what values of X is there an equilibrium in which players use all strategies with positive probability? We need to set the expected payoffs of A, B, and C equal to each other, which is possible only if $\sigma_1(L) = \frac{2}{3}$ and $X = 32$.