Economic Theory (2008) 35: 37–56
DOI 10.1007/s00199-007-0223-x

RESEARCH ARTICLE

Vlad Mares · Mikhael Shor

Industry concentration in common value auctions: theory and evidence

Received: 1 February 2006 / Revised: 23 January 2007 /
Published online: 7 March 2007

Abstract We examine theoretically and experimentally two countervailing effects of industry concentration in common value auctions. Greater concentration of information among fewer bidders reduces competition but increases the precision of private estimates. We demonstrate that this generally leads to more aggressive bidding. However, the reduction in competition dominates the informational effects, resulting in lower prices. We examine these hypothesized effects experimentally by conducting a series of auctions with constant informational content but distributed among a varying number of bidders. The experimental results are consistent with our theoretical predictions.

Keywords Common value auctions · Information · Joint bidding · Industry concentration

JEL Classification Numbers D44 · L41 · C92

The authors would like to thank Octavian Carare, Eric Friedman, Luke Froeb, Ron Harstad, Toshi Iizuka, Mike Rothkopf, Charles Thomas, and two anonymous referees, for many useful comments and suggestions.

V. Mares
Olin School of Business, Washington University,
St. Louis, MO, USA
E-mail: mares@wustl.edu

M. Shor
Owen Graduate School of Management,
Vanderbilt University, Nashville, TN, USA

M. Shor (✉)
352 Management Hall, 401 21st Avenue South,
Nashville, TN 37203, USA
E-mail: mike.shor@owen.vanderbilt.edu

1 Introduction

In common-value auctions, more accurate signals about the value of the item for sale alleviate the winner's curse, potentially leading to higher bids. But what if more accurate signals come at the expense of competition by concentrating existing information among fewer bidders? In determining auction revenue, more aggressive bidding is tempered by fewer tendered bids.

Consider, for example, an auction for an oil field for which six seismic surveys are available. For simplicity, assume that the surveys are correct on average—the average of the six surveys represents the true value of the field. What happens if we move from six bidders, each privy to the results of one survey, to three, each with two surveys in hand? First, possessing multiple surveys likely leads to a more informed estimate of the field's true value. This "information pooling" alleviates the winner's curse (DeBrock and Smith 1983; Hendricks and Porter 1992). Second, reducing the number of active bidders may, in itself, lead to higher bids (Pinkse and Tan 2005; Bulow and Klemperer 2002; Hendricks et al. 2003). Outbidding fewer competitors implies less chance that one's estimate is overly optimistic, decreasing the likelihood of overbidding. Lastly, the reduction in competition counteracts these forces, tending to lower the auction price.

We derive the impact of greater industry concentration on bids and revenues in these environments. We add to the existing literature on the average value model, in which an object's value is equal to the mean of all signals (Bikhchandani and Riley 1991; Krishna and Morgan 1997; Mares 2001; Bulow and Klemperer 2002), a comprehensive description of symmetric equilibrium bidding by parties with multiple signals, and a general result on the price impact of greater concentration. We extend some existing results for second-price auctions to first-price auctions. Better information generally leads to more aggressive bidding, though with caveats for first-price auctions. However, the impact of reduced competition outweighs the effects of better information, resulting in lower prices.

As in the above example, we take a view of greater concentration as the allocation of a fixed number of estimates of an object's value among fewer bidders. In his paper on information acquisition in auctions, Matthews (1984) provides a series of examples suggesting that the total amount of information bidders acquire may converge to a positive constant as the number of bidders increases. Matthews also points to serious technical difficulties of constructing full equilibrium models that endogenize information acquisition as a function of competitor and market variables. One way around this analytical difficulty is to fix the total quantity of information and let the concentration of that information vary with competition. This approach is mirrored in work by Krishna and Morgan (1997). A change in the concentration of information does not alter the *structure* of information but the *allocation* of that information among bidders. Industry and information concentration are inexorably linked; if bidders acquire more information in the presence of reduced competition, then fewer bidders compete in informationally more concentrated environments.

Our approach shares some similarities with models of horizontal merger waves. In many instances, industries are subject to a rapid series of acquisitions following exogenous shocks that affect valuations (Gort 1969; Jovanovic and Rousseau 2002). After a wave of acquisitions, merged entities face fewer competing bidders and are better informed by pooling the information of their constituents. Here, we treat such merger waves (and competition more generally) as exogenous, instead focusing on the equilibrium implications of increased information concentration in auctions.[1]

Laboratory experiments were run to test our theoretical predictions and evaluate the sensitivity of prices to information concentration. The growing body of experimental literature on common value auctions shows that people often fail to bid in accordance with equilibrium predictions and frequently fall prey to the winner's curse. Experiments using the average value model reveal overbidding by subjects in both first-price Holt and Sherman (2000) and second-price (Avery and Kagel 1997) auctions with two bidders. In both of these experiments, many subjects appeared to bid based on the unconditional expectation of rivals' signals, failing to account for the winner's curse. Goeree and Offerman (2002) provide bidders with two signals, one a common (average value) component and another an additive private value component. Again, subjects frequently lose money.

In the next section, we derive theoretical results on the impact of information and industry concentration on bids and revenues. Since we can expect human subjects to deviate from equilibrium bidding behavior, the subsequent section describes experiments testing the qualitative implications of our theoretical results. We find that comparative static results still obtain, as subjects bid higher in treatments reflecting more concentrated industries, though revenues nevertheless decline.

2 Theory

2.1 Model

We examine the average value model because it allows for closed-form solutions of the bidding function at various industry concentration profiles.[2] An object's value, V, is equal to the mean of all signals received by participants:

$$V = \frac{\sum_{i=1}^{n} X_i}{n}$$

Private signals, X_i, are *i.i.d.* with distribution function F and density f. We limit ourselves to situations where the support of the distribution is bounded, and for expositional simplicity, we assume that the support is [0, 1], although most results are easily extended to non-compact supports. Denote by $\beta_n^{FP,X}(x)$ and $\beta_n^{SP,X}(x)$ the symmetric equilibrium bids in first-price and second-price auctions with n bidders where signals are distributed *i.i.d.* as X.

[1] While stronger competition diminishes private incentives to acquire information, equilibrium effects of endogenizing collusion or mergers raise the additional complication of determining incentive-compatible mechanisms to elicit information within bidding groups. Mailath and Zemsky (1991) characterize a mechanism for collusive rings in a private-value auction to both elicit truthful revelation and allocate the object efficiently within the ring. Also see McAfee and McMillan (1992). In a model similar to ours, Brusco et al. (2006) examine the incentive to merge and consider conditions under which collusive rings can share information efficiently. Harford (2005) also considers the importance of informational acquisition concerns in merger decisions.

[2] See Krishna and Morgan (1997) and Mares (2001). Additive specifications have been used extensively in the literature (e.g., Bikhchandani and Riley 1991; Bulow and Klemperer 2002). Multi-signal environments are not merely more challenging than environments with one signal, but may not have *any* equilibria (Jackson 2005).

In this paper, we consider only symmetric information profiles in which each bidder sees the same number of signals. This avoids most complications arising from equilibrium selection problems[3] and facilitates the determination of equilibrium bids since the problem remains symmetric. An industry consists of m bidders receiving k signals each, where $n = km$ remains constant. Across concentration profiles, the total amount of information in the economy remains unchanged. Each bidder's private information is represented by $\overline{X}_k$, the random variable that is the average of k independent signals.

2.2 Results

A number of studies have identified effects of greater industry concentration in *second* price auctions described by this model in which fewer firms corresponds with each firm having more information. First, the aggregation of independent signals allows a bidding consortium to pool its information, deriving more precise estimates of the object's value and potentially implying a smaller winner's curse correction (DeBrock and Smith 1983; Krishna and Morgan 1997). Second, decreasing the number of bidders, even without an offsetting increase in information for each bidder, may lead to more aggressive bidding. In auctions with fewer bidders, subjects need not be as cautious to avoid the winner's curse. These effects tend to work in the same direction as better information coupled with a reduction in the number of bidders leads to higher bids, on average. In first-price auctions, an additional effect works in the opposite direction. When faced with fewer competitors, bidders can bid less aggressively and still maintain their chance of winning. This implies that the impact of greater concentration on bids in first-price auctions is ambiguous.[4]

In the following lemmas, we determine the role of each of these effects independently. The first isolates the role of the number of bidders without an offsetting increase in informational precision. The second considers the role of more precise information without a change in the number of bidders. We then show the overall effect of greater industry and information concentration on bidding behavior.

Competition Effect Symmetric equilibrium bids in an average value model with n bidders are given by[5]

$$\begin{aligned}\beta_n^{FP,X}(x) = {} & \frac{(n-1)}{n} E[X|X \leq x] \\ & + \frac{1}{n} E[\max(X_1, \ldots, X_{n-1})|\max(X_1, \ldots, X_{n-1}) < x], \qquad (1)\end{aligned}$$

[3] Second price common value auctions (both symmetric and asymmetric) may admit a continuum of equilibria (Bikhchandani and Riley 1991; Milgrom 1981), some yielding collusive-like outcomes. The symmetric equilibrium is a natural selection device both because it treats bidders anonymously and because it selects the "most competitive" equilibrium producing the highest revenue (Bikhchandani and Riley 1991; Mares 2005). Asymmetric situations lack an equally appealing equilibrium selection tool complicating revenue comparisons across auctions.

[4] Pinkse and Tan (2005) and Bulow and Klemperer (2002) demonstrate that bids may decline with more bidders. Hendricks et al. (2003) demonstrate this effect in OCS wildcat auctions. The opposite result has also been obtained experimentally (Kagel and Levin 1986) and in econometric analysis of field data (Hong and Shum 2002).

[5] These are obtained by substituting our model's value function into the formulation of Milgrom and Weber (1982).

$$\beta_n^{SP,X}(x) = \frac{2}{n}x + \frac{n-2}{n}E[X|X \leq x] \tag{2}$$

Increasing the number of bidders decreases bids in a second-price auction. Intuitively, the more people I outbid, the more likely it is that I have overestimated the object's value and fallen prey to the winner's curse; more bidders imply a greater adjustment for the winner's curse. Analysis of first-price auctions is less straightforward. However, the next result shows that for large enough n we find the same result as in the second-price auction. An increase in the number of participants produces less aggressive bidding.

Lemma 1 (Competition effect) *For every* $x \in [0, 1]$,

(i) $\beta_n^{FP,X}(x)$ *is unimodal in* n.
(ii) $\beta_n^{SP,X}(x)$ *is decreasing in* n.

The proofs of all results are in the appendix. In first-price auctions, bidders shade from the expected value not only to account for the winner's curse but also to balance the price paid with the odds of winning. Reducing the number of bidders encourages a smaller winner's curse adjustment as in second-price auctions but also implies more bid shading since any bid is more likely to win with fewer opponents. These effects work in opposite directions, though the impact of reducing the winner's curse dominates when the number of participants is sufficiently large.

Information Pooling In this section, we consider the effect of increasing the precision of bidders' information by providing each bidder with multiple signals while holding the number of bidders constant. How the equilibrium bidding functions, (1) and (2), respond to more information depends on comparisons between $E[X|X \leq x]$ and $E[\overline{X}_k|\overline{X}_k \leq x]$. If X has a log-concave density,[6] then we can establish the following:

Claim 1 Let f be log-concave. For $k \in \{1, 2, \ldots\}$ and for all x in $[0, 1]$,

$$E[\overline{X}_k|\overline{X}_k \leq x] \geq E[X|X \leq x]. \tag{3}$$

In second-price auctions, the equilibrium bid is a convex combination of one's own signal, x, and of $E[X|X \leq x]$, leading to higher bids as the precision of information increases. In first-price auctions, the role of more precise signals is less absolute, as the following lemma indicates.[7]

[6] The class of log-concave distributions encompasses most common distributions including the uniform, normal, logistic, exponential, and Weibull. For a comprehensive list of log-concave distributions and a review of their uses in auction theory, mechanism design, and other areas of economics, see Bagnoli and Bergstrom (2005).

[7] While our primary interest in the context of the average value model is in the distribution of $\overline{X}_k$, in the appendix we also prove an alternate form of Lemma 2 of more general interest. Consider two random variables X and Y with equal supports and identical first moments. Following Whitt (1985), Y is more precise than X in the sense of the log-concave order, denoted by $Y \preceq_{lc} X$, if $\frac{f_Y}{f_X}$ is log-concave. The log-concave order compares distributions in terms of their relative concentration around the mean and implies the usual (convex) order between random variables. Whitt (1985) obtains a result analogous to Claim 1: if $Y \preceq_{lc} X$, then $E[Y|Y \leq x] \geq E[X|X \leq x]$. Using this, we derive an analog to Lemma 2 for a distribution Y in place of $\overline{X}_k$.

Lemma 2 (Information pooling) *Let f be log-concave. Then*

(i) *there exist a $t_{k,n,F}$ and $t'_{k,n,F}$, $0 < t_{k,n,F} \leq t'_{k,n,F} < 1$, such that $\beta_n^{FP,\overline{X}_k}(x) \geq \beta_n^{FP,X}(x)$ for x in $[0, t_{k,n,F}]$ and $\beta_n^{FP,\overline{X}_k}(x) \leq \beta_n^{FP,X}(x)$ for x in $[t'_{k,n,F}, 1]$; and*
(ii) $\beta_n^{SP,\overline{X}_k}(x) \geq \beta_n^{SP,X}(x)$ *for all x.*

The lemma demonstrates that in first-price auctions, better information will increase bids for low signals while decreasing bids for high signals. To understand why this occurs, note that the first term in the equilibrium bidding function, (Eq. 1), increases if X is replaced with $\overline{X}_k$ by Claim 1. For high values of the signal, this effect diminishes since the unconditional expectations are equal, $E[X] = E[\overline{X}_k]$. Thus, for sufficiently high signals, the impact of more information is driven by the second term of the bidding function. Consider a bidder with a signal equal to 1, the upper support. If signals are more concentrated, having a competitor with a signal close to 1 is less likely than if the distribution were more dispersed. A bidder with a signal of 1 can profitably shade his bid more when he faces opponents with more concentrated information, leading to lower bidding.

Equilibrium Bidding Increased concentration gives rise to both the competition and information pooling effects since we are reducing the number of bidders while simultaneously increasing informational precision. Both effects lead to more aggressive bidding in second-price auctions while first-price auctions present some subtleties. The following result characterizes the overall impact of greater concentration on bids. We wish to compare an industry in which $n = km$ firms each have one signal with more concentrated industries in which each of $m < n$ firms has k signals. The more concentrated industry gives rise to an auction in which each bidder receives a signal $\overline{X}_k$. Denote by $\beta_{km}^{FP,X}(x)$ and $\beta_m^{FP,\overline{X}_k}(x)$ the symmetric equilibrium bid functions in the less and more concentrated industries in first-price auctions and by $\beta_{km}^{SP,X}(x)$ and $\beta_m^{SP,\overline{X}_k}(x)$ the bid functions in second-price auctions.

Theorem 1 *Let f be log-concave. Then*

(i) *For fixed k and high enough n, there exist a $s_{k,n,F}$ and $s'_{k,n,F}$, $0 < s_{k,n,F} \leq s'_{k,n,F} < 1$, such that $\beta_{km}^{FP,X}(x) \leq \beta_m^{FP,\overline{X}_k}(x)$ when (a) x in $[0, s_{k,n,F}]$, and (b) x in $[s'_{k,n,F}, 1]$; and*
(ii) $\beta_{km}^{SP,X}(x) \leq \beta_m^{SP,\overline{X}_k}(x)$ *for all x.*

The theorem states that greater industry concentration unambiguously increases the aggressiveness of bidding in second-price auctions. The result for first-price auctions incorporates the nuances of Lemmas 1 and 2. Greater concentration leads to more aggressive bidding for low values of the signal. For high signals, greater concentration may induce less aggressive bidding, but only when the amount of information in the economy is small. In particular, for any k, $\beta_{km}^{FP,X}(1) < \beta_m^{FP,\overline{X}_k}(1)$ for high enough n. For high values of the signal, a large amount of information in the economy (large n) implies that the information pooling effect outweighs the reduction in the number of bidders. The theorem allows for the possibility of a crossing between $\beta_{km}^{FP,X}$ and $\beta_{m}^{FP,\overline{X}_k}$, at least for low values of m. Our experimental set-up, with $n = 6$, and uniformly distributed signals, exhibits this property.

Equilibrium Prices Since bidding can be more aggressive as information becomes more concentrated, it is conceivable that greater industry concentration would lead to higher revenues. This conjecture was forwarded by Krishna and Morgan (1997) and echoed in the legal antitrust literature (Froeb and Shor 2005). We find, in our model, that the reduction in the number of bidders outweighs the increased aggressiveness of bids.

Theorem 2 *In the average value model, greater concentration reduces expected revenues.*

The result applies to both first-price and second-price auctions since, by our model's construction, revenue equivalence holds. Aggregating all the forces that affect equilibrium bidding, we find that the overall reduction in the number of bidders outweighs in equilibrium all gains that can be expected from more aggressive bidding. Unlike our results on the bidding function, we do not require distributional assumptions. Instead, the proof relies on properties of stochastic dominance and order statistics. The intuition behind this result is that a bidder's informational contribution is more significant in situations where concentration is higher. This means he can command a higher informational rent in any mechanism, thereby reducing revenues.

3 Experiments

Experimental subjects participated in a series of three auctions. In each auction, a total of six signals drawn from a discrete uniform distribution over the integers $\{20, 21, \ldots, 60\}$ were distributed among the bidders. The value of the object (in dollars) was known to be equal to the average of these signals. In the first period, each of six bidders received one signal, and placed a bid. In the second period, subjects competed against two other bidders (three total), with each participant receiving two signals. Lastly, period three saw two bidders competing with three signals each.

Each of 102 subjects was randomly assigned to either a first-price (60 subjects) or second-price (42 subjects) sealed-bid treatment (Table 1). Subjects submitted bids independently over the Internet and were not informed of the outcome of any auction until after the conclusion of the experiment. Subjects were told that they would not be bidding against the same bidder more than once and were randomly matched (while respecting this constraint) ex post, at which point earnings were tabulated.[8]

Experiments were run at Vanderbilt University in the Fall of 2001. Subjects were current MBA students and all had some classroom exposure to common value

[8] Subjects received a \$5 participation fee and participated in other experiments not reported here in which losses were not possible, to make up for potential losses from the winner's curse.

Table 1 Experimental treatments

Treatment		*m*	*k*	# of subjects	# of auctions
	I a	6	1		10
1st Price	I b	3	2	60	20
	I c	2	3		30
	II a	6	1		7
2nd Price	II b	3	2	42	14
	II c	2	3		21

auctions. In informal interviews after the experiment, most subjects indicated some experience with participating in auctions, ranging from low value online purchases to formulating bidding "strategies" for mid-size businesses in procurement auctions. Notably, none admitted to any experience with collusion (or joint bidding) in auctions.

3.1 Results

Both treatments demonstrate that concentration increases bids and decreases prices (Fig. 1). Specifically, the bidding functions increase with concentration (Fig. 2), yet this is not enough to offset the loss of competition. When six bidders with one signal each compete, bids significantly exceed theoretical predictions, in line with

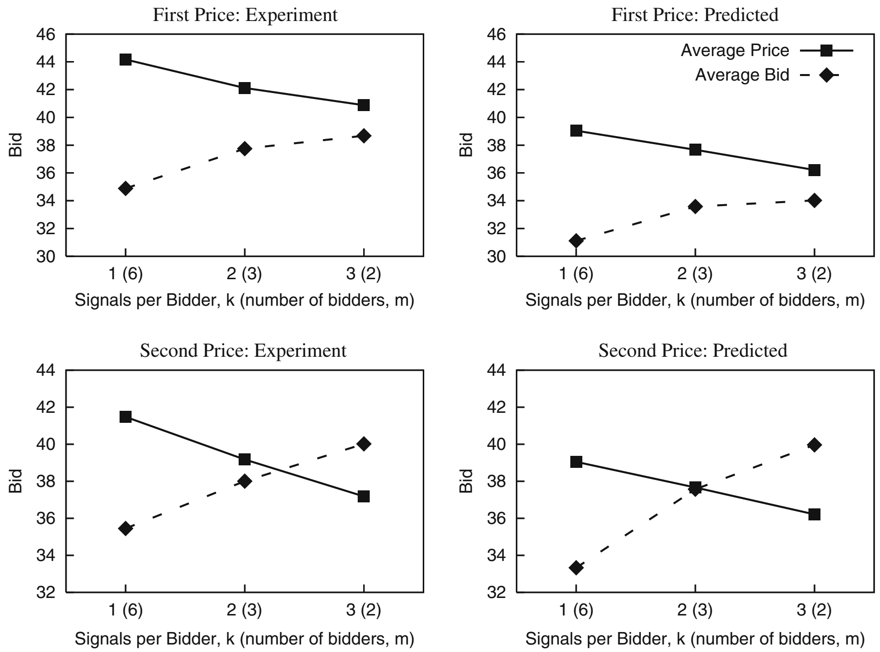

Fig. 1 Experimental and predicted average bids and prices. Greater concentration leads to higher bids on average but decreases the winning price

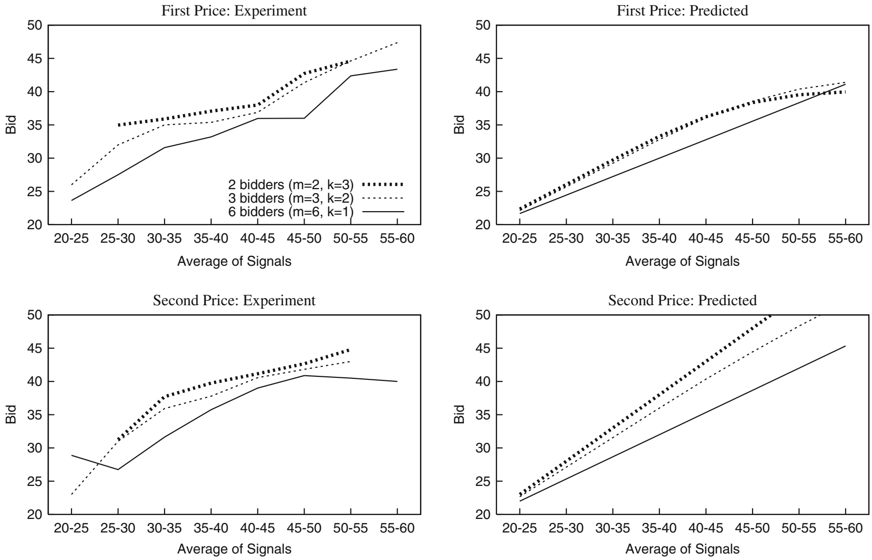

Fig. 2 Experimental and predicted bidding functions. In both first-price and second-price auctions, bidding becomes more aggressive with increased concentration

previous findings (Holt and Sherman 2000; Avery and Kagel 1997).[9] With multiple signals in the three and two bidder cases, bids exceed theoretical predictions only in first-price auctions.[10] Especially in second-price auctions, the bidding functions appear flatter than predicted, suggesting that subjects fail to account for the *conditional* expectation of rivals' signals.[11] While the trend in revenues appears in line with the theoretical results, the results of these experimental sessions are mostly anecdotal. A different random pairing of subjects would potentially produce different results. For example, two of the three highest bids in the first-price treatment were matched into the same group of six bidders. If these bidders were instead in different groups, both would likely win, increasing the average winning price.

Our interest, however, is not in the specific results of these sessions, but in the effect of information concentration on *expected* bids and revenue. From the perspective of the auctioneer, participants map a distribution of signals into a distribution of bids, and the auction maps these into a distribution of (first or second) order statistics from which the auctioneer draws his revenue. We formulate an analogous approach. First, we compute the empirical distribution of bids for each auction treatment. Then, we derive an empirical distribution of resulting prices. [12]

[9] Wilcoxon signed-rank test comparing bids to expected bids, given the signal. One-tailed $P < 0.001$.

[10] In first-price auctions, $P < 0.001$. In second-price auctions, $P = 0.06$ (three bidders) and $P = 0.48$ (two bidders).

[11] This is the first finding in both Holt and Sherman (2000) and Avery and Kagel (1997).

[12] Considering bids independent of signals is only valid if realized signals do not vary across treatments. Empirical distributions of signals in each treatment are neither significantly different from the uniform distribution (Kolmogorov–Smirnov P values between 0.22 and 0.87) nor from each other in pair-wise tests (P values between 0.49 and 1.00).

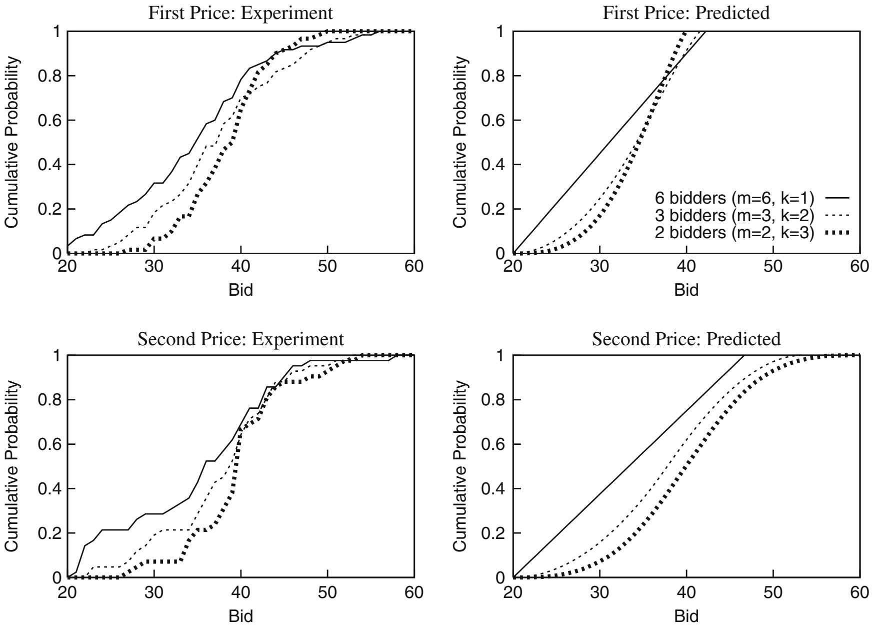

Fig. 3 Empirical and predicted cumulative distributions of bids

Bids are more concentrated about the mean bid as information is dispersed among fewer bidders (Fig. 3). This is hardly surprising since a more precise estimate of the object is obtained. More interesting results are obtained with respect to bidder aggressiveness.

Result 1 Bidding becomes more aggressive as information is concentrated from six bidders to two.

The empirical distribution of bids issued by participants with three signals (and one competitor) stochastically dominate bids by holders of one signal in the six bidder case.[13] For most bids (including those below the expected value of the object), the probability that the bidder bids higher than some given amount is larger in the two-bidder case. Incremental changes in concentration—from six bidders to three and from three to two—do not lead to significant differences in the distribution of bids. Graphically, bidding in the three-bidder case appears more aggressive than in the six-bidder case, and in fact average bids are significantly higher.[14]

The effect of concentration on bidding appears in line with Theorem 1. A greater concentration of signals among fewer bidders increases the aggressiveness of bids. However, the next result shows that this effect is tempered by the loss of competition.

[13] These distributions are significantly different. K–S $P = 0.013$ for first-price and $P = 0.028$ for second-price.

[14] Wilcoxon signed-rank test yields $P = 0.015$ for first-price and $P = 0.049$ for second-price, one tailed. Change in average bids from three to two bidders is marginally significant ($P = 0.074$) only in the second-price treatment. Differences from six bidders to two are highly significant ($P < 0.01$).

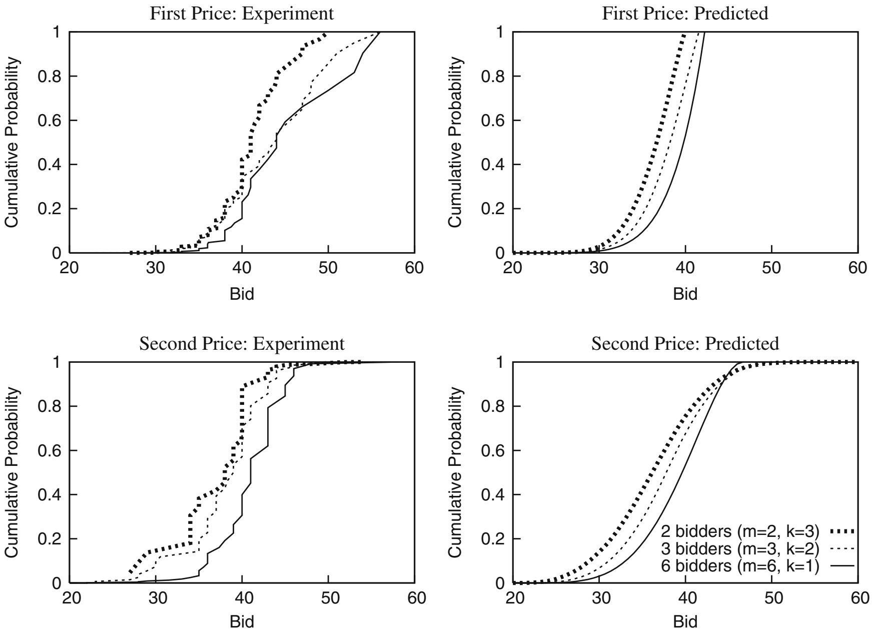

Fig. 4 Empirical and predicted cumulative distributions of revenue (winning bids)

Result 2 The expected auction price decreases with greater concentration.

This inference could be drawn from the observed winning bids (Fig. 1). The more general question is whether bidding behavior observed in this experiment implies higher prices *on average* rather than as the result of one pattern of random pairing of subjects. To answer this question, the empirical bid distributions were used to calculate the probability of a given auction price. For example, for the six-bidder (one signal each) first-price auction, the probability of a given price is equivalent to the probability that one bidder "draws" this value from the appropriate bid distribution and the remaining bidders draw smaller values. Even though subjects generally bid higher when information is more concentrated, this need not translate into higher revenues when the loss of competition is considered. The derived distributions of prices are presented in Fig. 4.

The distribution of prices in more information-concentrated auctions is dominated by those of less-concentrated treatments.[15] For almost any target price, bidders in more concentrated industries are less likely to meet the target than bidders in less concentrated industries.[16] Theoretically predicted revenues for the six-,

[15] Kolmogorov-Smirnov test statistic in all pair-wise comparisons of distributions yields P-values of less than 0.001.

[16] We also compare average revenues to theoretical benchmarks. In two and three bidder cases, each possible grouping of subjects is considered. In six-bidder cases, 100*s* samples of groups are drawn, where s is the number of subjects (60 or 42 for first- or second-price autions). For each sampled group, resulting price is compared to expected price, given the signals. Because revenues of two samples are correlated if they share a subject, the recombinant estimator was used (Mullin and Reiley 2006). Results follow the pattern of bids. Observed revenues are significantly higher than predicted in first-price auctions ($P < 0.001$, one-tailed). In second-price auctions, results are mixed ($P = 0.001$, six bidders; $P = 0.057$, three bidders, and $P = 0.117$, two bidders).

three-, and two-bidder cases are 39.05, 37.67, and 36.20 respectively. These reflect decreases in revenue due to concentration of 3.5 and 9.3%. In the first-price auction with six bidders, the expected auction price derived from experimental data is 45.78. This decreases by 3.8% if information is concentrated among three bidders and by 10.0% if two bidders each receive three signals. In the second-price auction, the revenue effects are more pronounced, with reductions of 7.1 and 11.5% from the six-bidder revenue of 41.25.

4 Conclusion

Greater information concentration increases the aggressiveness of bidding in second-price auctions, reflecting the role of better information in diminishing the winner's curse. In first-price auctions, this effect is tempered by less bid-shading in the presence of fewer bidders. Yet, this more aggressive bidding does not offset the downward price pressures of diminished competition. Both theoretical results and experiments confirm that greater industry concentration leads to lower prices.

Antitrust policy is driven by an almost universally accepted maxim among economists: in the absence of offsetting efficiency gains, greater industry concentration decreases competition, leading to higher market prices. This traditional industrial organization analysis, exploiting the robust relationship between market concentration and industry performance, is directly applicable to private-value auction markets. For example, mergers among parties are privately profitable (Mailath and Zemsky 1991) and reduce the number of active bidders, leading to diminished revenue for the seller (Waehrer and Perry, 2003; Tschantz et al. 2000). Common-value auctions have called into question the causal relationship between market structure and efficiency, driven by a number of interdependent effects which make the consequences of mergers, collusion, and concentration generally, unclear. Legal antitrust thinking has advocated a hands-off approach to regulating common value auctions until these effects are better understood (Froeb and Shor 2005). Our results are in line with the traditional thinking about the effects of concentration on prices. It may very well be appropriate to extend the antitrust bias against greater concentration to common-value auction markets, though much more exploration is required.

Our design addresses the issue of concentration, generally, abstracting from several real issues of collusion and mergers. First, we assume that a merged consortium's representative has access to all of the members' signals and ignore the very real issues of designing incentive-compatible mechanisms for eliciting signals and sharing revenue among colluders. Our framework is more akin to a takeover scenario since the acquiring firm has an incentive to aggregate all available information. Second, the partition of signals is exogenous. Issues of incentives to merge and collude are therefore avoided. In this sense, the theoretical findings and experiment answer the "what if" effect of concentration, rather than the "why." Alternately, they may reflect mergers among multi-product firms for whom these auctions represent minor portions of revenue, and hence the effects of merging in this auction market are negligible relative to other concerns.

A first extension is to consider incentives to merge. Such analysis could have a prescriptive purpose, aiding our understanding of merger "waves," as firms not yet part of a bidding ring may perceive their inferior information as a competitive disadvantage. A second extension may focus on multi-unit auctions in which the informational effects must also be balanced against demand reduction.

Appendix

For the proofs of the lemmas, we require three technical results.

Proof of Claim 1 For independent signals with log-concavely density functions, $\frac{X_1+\cdots+X_k}{k}$ and $\max(X_1, \ldots, X_k)$ are affiliated. Hence,

$$\begin{aligned} &E[\overline{X}_k|\overline{X}_k \leq x] \\ &\quad \geq E[\overline{X}_k|\overline{X}_k \leq x, \max(X_1, \ldots, X_k) \leq x] \\ &\quad = E[\overline{X}_k|\max(X_1, \ldots, X_k) \leq x] \\ &\quad = E[X|X \leq x]. \end{aligned}$$

□

Claim 2 If F and G are positive continuous functions such that $[F(x)/G(x)]$ is decreasing (increasing) in x, then

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} \geq \frac{F(x)}{G(x)},$$

and $\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}$ is decreasing (increasing) in x.

Proof We will provide a proof only for the decreasing case. The statement for increasing can be obtained analogously. By Lagrange's theorem, there exists $\xi \in [0, x]$ such that

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} = \frac{F(\xi)}{G(\xi)} \geq \frac{F(x)}{G(x)}.$$

This property implies that $\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}$ is decreasing in x, since there exists an $\eta \in [x, x+y]$

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} \geq \frac{F(x)}{G(x)} \geq \frac{\int_x^{x+y} F(s)ds}{\int_x^{x+y} G(s)ds} = \frac{F(\eta)}{G(\eta)},$$

and hence

$$\frac{\int_0^{x+y} F(s)ds}{\int_0^{x+y} G(s)ds} = \frac{\int_0^x F(s)ds + \int_x^{x+y} F(s)ds}{\int_0^x G(s)ds + \int_x^{x+y} G(s)ds} \leq \frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}.$$ □

Claim 3 Let f_X be a log-concave density with support $[0, 1]$. Consider $k \in \{2, 3, \ldots\}$. There exists a $t > 0$, such that $\frac{F_{\overline{X}_k}}{F_X}$ is increasing on $[0, t]$.

Proof We will establish a slightly stronger result. By Claim 2, if $\frac{f_{\overline{X}_k}}{f_X}$ is increasing on some interval $[0, t]$ then so is $\frac{F_{\overline{X}_k}}{F_X}$. We will identify a $t > 0$, such that, $\frac{f_{\overline{X}_k}}{f_X}$ is increasing on $[0, t]$.

Log concave density implies that f_X is unimodal. First, consider the case where f_X is everywhere decreasing. Since it is a convolution of log-concave densities, $f_{\overline{X}_k}$ is also log-concave. This implies that there exists $t > 0$, such that $f_{\overline{X}_k}$ does not change monotonicity on $[0, t]$. Since $f_{\overline{X}_k}(0) = 0$, $f_{\overline{X}_k}$ is increasing on $[0, t]$, which means that $\frac{f_{\overline{X}_k}}{f_X}$ is increasing on $[0, t]$.

Next, consider the case where f_X is increasing on $[0, t']$ for some $t' > 0$. For simplicity, we let $k = 2$, though other cases are obtained analogously. For sufficiently small $y > x$,

$$\frac{f_{\overline{X}_2}(x)}{f_X(x)} = \int_0^{2x} \frac{f_X(2x - s) f_X(s)}{f_X(x)} ds = 2 \int_x^{2x} \frac{f_X(2x - s) f_X(s)}{f_X(x)} ds$$

due to the symmetry of the integrand around x. Because we ar elimiting ourselves to the region where $f_X(x)$ is increasing, we have $f_X(y - x + s) \geq f_X(s)$, implying

$$\leq 2 \int_x^{2x} \frac{f_X(2x - s) f_X(y - x + s)}{f_X(x)} ds$$

The log-concavity assumption implies that $\frac{f_X(y)}{f_X(x)} \leq \frac{f_X(y-(s-x))}{f_X(x-(s-x))}$ for $x \leq s \leq 2x$. Thus,

$$\leq 2 \int_x^{2x} \frac{f_X(y + x - s) f_X(y - x + s)}{f_X(y)} ds$$

when y is sufficiently small ($y < t'/2$). By changing the integration variable $s = r - (y - x)$,

$$= 2 \int_y^{y+x} \frac{f_X(2y - r) f_X(r)}{f_X(y)} dr \leq 2 \int_y^{2y} \frac{f_X(2y - r) f_X(r)}{f_X(y)} dr$$

$$= \int_0^{2y} \frac{f_X(2y - r) f_X(r)}{f_X(y)} dr = \frac{f_{\overline{X}_2}(y)}{f_X(y)}.$$

□

Proof of Lemma 1 (i) We proceed in two steps. First, we establish that if $X_1, \ldots, X_l$ are i.i.d. variates and if $k \leq l$, then

$$E[\max(X_1, \ldots, X_k) | \max(X_1, \ldots, X_k) \leq x] \leq E[\max(X_1, \ldots, X_l) | \max(X_1, \ldots, X_l) \leq x]$$

for all x. Since $\max(X_1, \ldots, X_k) \sim F^k$ and $\max(X_1, \ldots, X_l) \sim F^l$, Claim 2 implies that $\frac{\int_0^x F^k(s)ds}{\int_0^x F^l(s)ds}$ is decreasing. Taking the derivative of the logarithm of this expression yields:

$$\frac{\int_0^x F^l(s)ds}{F^l(x)} \leq \frac{\int_0^x F^k(s)ds}{F^k(x)} \quad \text{and} \quad x - \frac{\int_0^x F^k(s)ds}{F^k(x)} \leq x - \frac{\int_0^x F^l(s)ds}{F^l(x)}$$

which establishes the first step. Next, we show that $\beta_{n+1}^{FP,X}(x) - \beta_n^{FP,X}(x)$ crosses 0 only once.

$$\begin{aligned}\beta_{n+1}^{FP,X}(x) - \beta_n^{FP,X}(x) = &\frac{1}{n(n+1)} E[X|X \leq x] \\ &+ \frac{1}{n+1} E[\max(X_1, \ldots, X_n)| \max(X_1, \ldots, X_n) \leq x] \\ &- \frac{1}{n} E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) \leq x]\end{aligned}$$

Denote by $\mu_{k:n}^X$ the expected value of the kth lowest order statistic out of a sample of n i.i.d. random variables distributed as X. Define the random variable $Z = X|X \leq x$.

$$\begin{aligned}\beta_{n+1}^{FP,X}(x) - \beta_n^{FP,X}(x) &= \frac{1}{n(n+1)} \mu_{1:1}^Z + \frac{1}{n+1} \mu_{n:n}^Z - \frac{1}{n} \mu_{n-1:n-1}^Z \\ &= \frac{1}{n} \left(\mu_{n:n}^Z - \mu_{n-1:n-1}^Z\right) + \frac{1}{n(n+1)} \left(\mu_{1:1}^Z - \mu_{n:n}^Z\right)\end{aligned}$$

First, note that when $n = 2$, the above reduces to $\frac{1}{3}(\mu_{2:2}^Z - \mu_{1:1}^Z) > 0$. Second, the sequence $\mu_{n:n}^Z$ is increasing and tends tends to x. It is also well established that $\mu_{n:n}^Z - \mu_{n-1:n-1}^Z$ is decreasing in n. These facts imply that the expression above is decreasing, and negative for sufficiently large n.
(ii) Follows from Equation (2) and the fact that $E[X|X \leq x] \leq x$. □

Proof of Lemma 2 (i) Define the random variable $Y \equiv \overline{X}_k$ and denote by F and G the cumulative distribution functions of X and Y. By Claim 3, $\frac{G}{F}$ is increasing on $[0, \hat{t}]$ for some $\hat{t} > 0$. Following reasoning similar to Claim 2, we infer that $\frac{\int_0^x G^{n-1}(s)ds}{\int_0^x F^{n-1}(s)ds}$ is also increasing on $[0, \hat{t}]$. This implies

$$\begin{aligned}&E[\max(Y_1, \ldots, Y_{n-1})| \max(Y_1, \ldots, Y_{n-1}) \leq x] \\ &\quad \geq E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) \leq x]\end{aligned}$$

for all x in $[0, \hat{t}\,]$. Claim 1 also guarantees that $E[Y|Y \leq x] \geq E[X|X \leq x]$ for all x. This establishes the existence of $t_{k,n,F} \geq \hat{t}$ with the desired property. To demonstrate the existence of $t'_{k,n,F}$, we observe that the function $max : [0, 1]^n \to \mathbb{R}$ is convex in every component. Since Y is smaller than X in the convex order, we have, for all $n \geq 2$, $E[\max(Y_1, \ldots, Y_n)] < E[\max(X_1, \ldots, X_n)]$ (Shaked and Shanthikumar 1994).

$$\beta_n^{FP,Y}(1) = \frac{n-1}{n}E[Y] + \frac{1}{n}E[\max(Y_1, \ldots, Y_{n-1})]$$
$$< \frac{n-1}{n}E[X] + \frac{1}{n}E[\max(X_1, \ldots, X_{n-1})] = \beta_n^{FP,X}(1)$$

establishing the existence of $t'_{k,n,F}$ with the desired properties.
(ii) Follows directly from Claim 1. □

The uniform distribution, as in our experiments, yields the special case, $t_{k,n,F} = t'_{k,n,F}$. If the distribution is symmetric, one can verify that $t_{k,n,F} \geq E[X]$.

We also prove an alternate version of Lemma 2 for the log-concave order.

Lemma 3 (Lemma 2 for the log-concave order) *If $E[X] = E[Y]$ and Y is more precise than X in the log-concave order, $Y \preceq_{lc} X$, then*

(i) *there exist a $t_{X,Y,n}$ and $t'_{X,Y,n}$, $0 < t_{X,Y,n} \leq t'_{X,Y,n} < 1$, such that $\beta_n^{FP,Y}(x) \geq \beta_n^{FP,X}(x)$ for x in $[0, t_{X,Y,n}]$ and $\beta_n^{FP,Y}(x) \leq \beta_n^{FP,X}(x)$ for x in $[t'_{X,Y,n}, 1]$; and*
(ii) *$\beta_n^{SP,Y}(x) \geq \beta_n^{SP,X}(x)$ for all x.*

Proof The log-concave order implies that $\frac{G}{F}$ is unimodal and following reasoning similar to Claim 2, we infer that $\frac{\int_0^x G^{n-1}(s)ds}{\int_0^x F^{n-1}(s)ds}$ is unimodal and is increasing on $[0, \hat{t}]$. By Whitt (1985), $Y \preceq_{lc} X$ implies that $E[Y|Y \leq x] \geq E[X|X \leq x]$ for all x. Lastly, $Y_i \preceq_{lc} X_i$ implies that Y is smaller than X in the convex order. We can now parallel the steps of Lemma 2 to establish the desired result.

Proof of Theorem 1 For part (*i-a*), Lemma 1 implies that $\beta_m^{FP,\overline{X}_k}(x) \geq \beta_{km}^{FP,\overline{X}_k}(x)$ for all x when m is sufficiently large, and Lemma 2 provides that $\beta_{km}^{FP,\overline{X}_k}(x) \geq \beta_{km}^{FP,X}(x)$ for sufficiently small x. For (*i-b*), observe that the equilibrium bids are

$$\beta_{km}^{FP,X}(1) = \frac{km-1}{km}E[X] + \frac{1}{km}E[\max(X_1, \ldots, X_{km})], \text{ and}$$
$$\beta_m^{FP,\overline{X}_k}(1) = \frac{m-1}{m}E[\overline{X}_k] + \frac{1}{m}E[\max(\overline{X}_{k,1}, \ldots, \overline{X}_{k,m})].$$

By definition, $E[X] = E[\overline{X}_k]$. Note that for large m we have the following relationships

$$\left|E[\max(\overline{X}_{k,1}, \ldots, \overline{X}_{k,m})] - E[\max(X_1, \ldots, X_{km})]\right| \leq \varepsilon$$

for ε arbitrarily small since

$$\lim_{m\to\infty} E[\max(\overline{X}_{k,1}, \ldots, \overline{X}_{k,m})] = \lim_{m\to\infty} E[\max(X_1, \ldots, X_{km})] = 1.$$

This means that we can find m_k such that for $m > m_k$

$$E[\max(\overline{X}_{k,1}, \ldots, \overline{X}_{k,m})] > \frac{(k-1)}{k}E[X] + \frac{1}{k}E[\max(X_1, \ldots, X_{km})]$$

since as before we have that

$$E[\max(X_1, \ldots, X_{km})] > E[\max(\overline{X}_{k,\,1}, \ldots, \overline{X}_{k,\,m})] > E[X].$$

But this also establishes that for $m > m_k$

$$\begin{aligned}\beta_{km}^{FP,X}(1) &= \frac{m-1}{m}E[X] + \frac{1}{m}\left(\frac{(k-1)}{k}E[X] + \frac{1}{k}E[\max(X_1, \ldots, X_{km})]\right)\\ &< \frac{m-1}{m}E[X] + \frac{1}{m}E[\max(\overline{X}_{k,\,1}, \ldots, \overline{X}_{k,\,m})]\\ &= \beta_m^{FP,\overline{X}_k}(1).\end{aligned}$$

For part (ii), we have $\beta_{km}^{SP,X}(x) \leq \beta_m^{SP,X}(x) \leq \beta_m^{SP,\overline{X}_k}(x)$ where the first inequality follows from Lemma 1 and the second from Lemma 2. □

Theorem 1 allows for the possibility of a crossing between the two bidding functions, at least for low values of m. In the proof of Theorem 2 we make use of the following claim.

Claim 4 In the average value model, revenue from the symmetric equilibrium may be expressed as

$$R^F(n) = \mu^F - (\mu_{n:n}^F - \mu_{n-1:n-1}^F).$$

Proof Since revenue equivalence applies to this model, we need to present a proof only for second-price auctions. The symmetric equilibrium bid is

$$\beta_n^{SP,X}(x) = \frac{2}{n}x + \frac{n-2}{n}E[X|X \leq x] = x - \frac{n-2}{n}\frac{\int_0^x F(s)ds}{F(x)}.$$

The revenue, given distribution F, can be computed as

$$R^F(n) = \int_0^\infty \beta_n^{SP,X}(x) f_{n-1:n}(x)dx.$$

We will make use of the following recurrence relationship between the expectations for order statistics

$$n\mu_{n-1:n-1}^F = \mu_{n-1:n}^F + (n-1)\mu_{n:n}^F.$$ [17]

Also note that the density of the second highest order statistic out of a draw of n $i.i.d.$ variates is

$$f_{n-1:n}(x) = n(n-1)f(x)F^{n-2}(x)(1-F(x)).$$

[17] See, for example, Arnold et al. (1992).

Using the above relationships we can establish using integration by parts

$$
\begin{aligned}
R^F(n) &= \mu^F_{n-1:n} - \int_0^\infty (n-1)(n-2)\left(\int_0^x F(s)ds\right) f(x)F^{n-3}(x)(1-F(x))dx \\
&= \mu^F_{n-1:n} - \int_0^\infty \left(\int_0^x F(s)ds\right) f_{n-2:n-1}(x)dx \\
&= \mu^F_{n-1:n} - \int_0^\infty F(x)\,(1-F_{n-2:n-1}(x))\,dx \\
&= \mu^F_{n-1:n} - \int_0^\infty F(x) - (n-1)F^{n-1} + (n-2)F^n dx \\
&= \mu^F_{n-1:n} + \mu_F - (n-1)\mu^F_{n-1:n-1} + (n-2)\mu^F_{n:n} \\
&= \mu^F - (\mu^F_{n:n} - \mu^F_{n-1:n-1}).
\end{aligned}
$$

□

Proof of Theorem 2 We compare the case where $n = 2m$ bidders each receive 1 signal with the case where m bidders each receive 2 signals. Similar arguments establish the result for other $k \geq 2$. Let G be the distribution of the average of two private signals. According to Claim 4, revenues in these two cases are given by

$$
\begin{aligned}
R^G(m) &= \mu^G - (\mu^G_{m:m} - \mu^G_{m-1:m-1}) \\
&= \int_0^\infty \big(1 - G(s) - (1 - G^m(s)) + 1 - G^{m-1}(s)\big)\, ds \\
&= \int_0^\infty (1 - G(s))(1 - G^{m-1}(s))ds \\
R^F(2m) &= \int_0^\infty \big(1 - F(s))(1 - F^{2m-1}(s)\big)\, ds.
\end{aligned}
$$

Note also that by standard stochastic variability order arguments,[18] we can establish that

$$\int_0^x (1 - F(s))ds \geq \int_0^x (1 - G(s))ds \tag{4}$$

for every x. Since, by definition, we have that $\mu^F = \mu^G$, we can conclude that

$$R^F(2m) \geq R^G(m) \Leftrightarrow \int_0^\infty F^{2m-1}(s)(1 - F(s))ds \leq \int_0^\infty G^{m-1}(s)(1 - G(s))ds.$$

[18] See for example Shaked and Shanthikumar (1994).

We will establish the latter inequality in two steps

$$\int_0^\infty F^{2m-1}(s)(1 - F(s))ds \leq \int_0^\infty F^{2m-1}(s)(1 - G(s))ds$$

and

$$\int_0^\infty F^{2m-1}(s)(1 - G(s))ds \leq \int_0^\infty G^{m-1}(s)(1 - G(s))ds.$$

The last step is apparent once we observe that $\max(X_1, X_2) \geq \frac{X_1+X_2}{2}$, and since $\max(X_1, X_2) \sim F^2$ and $\frac{X_1+X_2}{2} \sim G$ we have by stochastic dominance that $F^2 \leq G$ and hence

$$F^{2m-1} \leq G^{m-1}$$

which gives us the desired second inequality.

For the first step, we can treat $1-F$ and $1-G$ (if necessarily normalized by μ^F and μ^G) as probability densities. The inequality in (4) establishes the stochastic dominance relationship between these two densities. Since F^{2m-1} is increasing, we obtain the desired inequality in step 1. □

References

Arnold, B. C., Balakrishnan, N., Nagaraja, H. N.: A First Course in Order Statistics. New York: Wiley (1992)

Avery, C., Kagel, J. H.: Second-price auctions with asymmetric payoffs: An experimental investigation. Econ Manage Strat **6**(3), 573–603 (1997)

Bagnoli, M., Bergstrom, T.: Log-concave probability and its applications. Econ Theory **26**(2), 445–469 (2005)

Bikhchandani, S., Riley, J.: Equilibria in open common value auctions. J Econ Theory **53**(1), 101–130 (1991)

Brusco, S., Lopomo, G., Robinson, D. T., Viswanathan, S.: Efficient mechanisms for mergers and acquisitions. Int Econ Rev (in press) (2006)

Bulow, J., Klemperer, P.: Prices and the winner's curse. RAND J Econ **33**(1), 1–21 (2002)

DeBrock, L., Smith, J.: Joint bidding, information pooling, and the performance of petroleum lease auctions. Bell J Econ **14**(2), 395–404 (1983)

Froeb, L., Shor, M.: Auction models. In: Harkider, J. D., (ed.) Econometrics: Legal, Practical, and Technical Issues, pp. 225–246. American Bar Association Section of Antitrust Law (2005)

Goeree, J. K., Offerman, T.: Efficiency in auctions with private and common values: An experimental study. Am Econo Rev **93**(3), 625–643 (2002)

Gort, M.: An economic disturbance theory of mergers. Q J Econ **83**(4), 624–642 (1969)

Harford, J.: What drives merger waves? J Financ Econ **77**(3), 529–560 (2005)

Hendricks, K., Pinkse, J., Porter, R. H.: Empirical implications of equilibrium bidding in first price common value auctions. Rev Econ Stud **70**(1), 115–145 (2003)

Hendricks, K., Porter, R. H.: Joint bidding in federal ocs auctions. Am Econ Rev **82**(2), 506–511 (1992)

Holt, C. A., Sherman, R.: Risk aversion and the winner's curse. Working Paper, University of Virginia (2000)

Hong, H., Shum, M.: Increasing competition and the winner's curse: Evidence from procurement. Rev Econ Stud **69**(4), 871–898 (2002)

Jackson, M. O.: The non-existence of equilibrium in Vickrey, second-price, and english auctions. Working Paper, California Institute of Technology (2005)

Jovanovic, B., Rousseau, P. L.: The q-theory of mergers. Am. Econ. Rev. **92**(2), 198–204 (2002)

Kagel, J. H., Levin, D.: The winner's curse and public information in common value auctions. Am Econ Rev **76**(5), 894–920 (1986)

Krishna, V., Morgan, J.: (anti-) competitive effects of joint bidding and bidder restrictions. Working Paper, Penn State University and Princeton University (1997)

Mailath, G. J., Zemsky, P.: Collusion in second price auctions with heterogeneous bidders. Games Econ Behav **3**(4), 467–486 (1991)

Mares, V.: Essays in Joint Bidding. Doctoral dissertation, Rutgers University (2001)

Mares, V.: Monotonicity and selection in second price auctions. Working Paper, Washington University, St. Louis (2005)

Matthews, S. A.: Information acquisition in discriminatory auctions. In: Boyer, M., Kihlstrom, R. E.: (eds.) Bayesian Models in Economic Theory, pp. 181–207. Elsevier Science: Amsterdam (1984)

McAfee, R. P., McMillan, J.: Bidding rings. Am Econ Rev **82**(3), 579–599 (1992)

Milgrom, P.: Rational expectations, information acquisition, and competitive bidding. Econometrica **49**(4), 921–943 (1981)

Milgrom, P., Weber, R.: A theory of auctions and competitive bidding. Econometrica **50**(5), 1089–1122 (1982)

Mullin, C. H., Reiley, D. H.: Recombinant estimation for normal-form games, with applications to auctions and bargaining. Games Econ Behav **54**(1), 159–182 (2006)

Pinkse, J., Tan, G.: The affiliation effect in first-price auctions. Econometrica **73**(1), 263–277 (2005)

Shaked, M., Shanthikumar, J. G.: Stochastic Orders and Their Applications. New York: Academic (1994)

Tschantz, S., Crooke, P., Froeb, L.: Mergers in sealed vs. oral auctions. Int J Econ Bus **7**(2), 201–213 (2000)

Waehrer, K., Perry, M. K.: Mergers in auction markets. RAND J Econ **34**(2), 287–304 (2003)

Whitt, W.: Uniform conditional variability ordering of probability distributions. J Appl Prob **22**(3), 619–633 (1985)