Mergers in Auctions with an Incumbent Advantage

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Abstract

When the winner of one auction gains a cost advantage in the next, bids reflect not only the value of winning the auction, but also the value of gaining an incumbent advantage in future auctions. In this setting, each firm faces an inter-temporal trade-off between 'harvesting' incumbency status by bidding less aggressively, and 'sowing' incumbency for the next period by bidding more aggressively. Mergers affect this tradeoff and change bids for merging as well as non-merging firms. We find that the presence of incumbent advantage leads to substantially different predictions about the competitive effects of mergers than those obtained in the absence of incumbent advantage. In particular, mergers are likely to be pro-competitive when the merging firms are relatively small, with fewer non-merging bidders, and when the incumbent advantage is large.

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JEL Codes: D44 (auctions), C72 (Dynamic Games), L41 (Horizontal Anticompetitive Practices)

1 Introduction

In repeated procurement auctions, the winner of one auction may gain a cost advantage in the next. For example, an accounting firm may save on audit costs if it was also the auditor in the previous period. Such an "incumbent advantage" may be due to learning or sunk-cost expenditures and is present in industries ranging from road building (Silva, Dunne and Kosmopoulou 2003) and business insurance (Opticare *v.* Marsh & McLennan 2004) to mainframe computers (Greenstein 1993), IT projects, and mobile telephony (Klemperer 2007). We analyze competition in this environment in order to better understand the effects of mergers.

To isolate the the effect of incumbent advantage on mergers, we consider the simplest of auction formats: a second-price, private-value auction. When there is no incumbent advantage, firms' dominant strategy to bid one's cost is simply repeated from auction to auction. Thus, mergers affect price only when the merged coalition contains the two lowest-cost bidders. When this happens, a merger eliminates competition between the merging bidders and causes price to increase from the second- to the third-lowest cost. Absent efficiency gains, mergers always raise expected price, but do not otherwise affect bidding by non-merging firms (e.g., Froeb, Tschantz and Crooke 2001). This merger characterization has been used by antitrust enforcement agencies to quantify the price effects of mergers between hospitals, mining equipment companies, and defense contractors, among others (Baker 1997).

With an incumbent advantage, however, bidders face a strategic dependency similar in complexity to that between bidders in a first-price auction. In particular, each firm faces an intertemporal trade-off between 'harvesting' incumbency status by bidding less aggressively, and 'sowing' incumbency for the next period by bidding more aggressively. Since this tradeoff depends on rival bids, analytical closed-form bidding functions are available only in highly stylized cases with a finite number of bidder types bidding in a finite number of auctions (e.g., Plum 1992, Lebrun 1999, Maskin and Riley 2000, Arozamena and Cantillon 2004, Kaplan and Zamir 2007).

In these dynamic models, industry performance differs from that in comparable static models. For example, Tang Sørensen (2006) and von der Fehr and Riis (2000) consider a two-period n-bidder second-price auction in which bidders do not learn their values in the second period until after the first auction, and the distribution of the winner's value is different from that of the losers. Prices may rise or fall from the first to the second period, and are not necessarily monotonic in the number of bidders. Leufkens, Peeters and Vermeulen (2008) examine sequential bidding on two contracts where winning the first increases the value of the second. Bidders' profits are decreasing in the size of the value advantage due to much more aggressive bidding in the first round. Jeitschko and Wolfstetter (2002) and Leufkens and Peeters (2007) use the results of Plum (1992) to solve for the equilibrium in a two-period two-bidder model. Bidding in the first period is more aggressive than in the second, as bids reflect not only the value of winning the auction, but also the value of becoming the incumbent in the second period. In this setting, they find that the second-price auction revenue dominates the first-price auction.

Neither the first- or second-price auction is optimal in these environments because the auctioneer does not adjust to the asymmetry created by incumbent advantage. Nonetheless, we follow this approach not only because it allows direct comparisons to past studies but more importantly because it corresponds to practice. In government procurement, for example, it is not uncommon (and often required by law) for auctioneers to ignore the asymmetries that arise from incumbency.[1] Despite their theoretical advantages (Brannman and Froeb 2000), governments generally do not use bidder-specific asymmetric auctions due to the heavy informational requirements and risk of political opportunism (Appelman, Gorter, Lijesen, Onderstal and Venniker 2003).

We depart from previous approaches in two ways. First, we use an infinite-horizon setting where bidders account for the value of future incumbency in *each* period. This allows us to get rid of the unrealistic "final" period where concerns about the the future disappear and it becomes optimal to "harvest" incumbency status by bidding less aggressively. Second, we allow for a richer characterization of asymmetry by using the logit auction model.[2] This is important because our application requires three types of asymmetry: the asymmetry created by merger, since even a symmetric industry will become asymmetric post-merger; the asymmetry that arises from incumbent advantage; and the *a priori* asymmetry among firms that arises naturally in most industries. The cost of this modeling complexity is that we sacrifice analytic closed-forms in favor of numerical solutions, and gain insight into the model predictions through a series of computational experiments. The experiments are designed to answer the policy question that motivates our interest in the topic: how should the competition agencies analyze mergers among bidders in auctions with an incumbent advantage?

We find that the presence of incumbent advantage leads to substantially different predictions about the competitive effects of mergers than those obtained in the absence of incumbent advantage. In the presence of an incumbent advantage, a firm with an *ex ante* advantage has a greater chance of retaining (and thus places a greater value on obtaining) incumbency which causes it to bid more aggressively. As a consequence, mergers that create firms with a bigger *ex ante* advantage cause more aggressive bidding. Whether this leads to lower prices depends also on the non-merging firms, whose bids set the price when the merged firm wins. Larger non-merging firms bid more aggressively, and this tends to mitigate the loss of competition caused by merger. We conclude that mergers are likely to be pro-competitive when the merging firms are relatively small, with fewer non-merging bidders, and when the incumbent advantage is large.

[1]See, for example, Porter and Zona (1993) and Jofre-Bonet and Pesendorfer (2000) in the context of highway paving contracts. Also, the European Union's public procurement regulations do not permit unequal treatment of incumbents (EU 2004). McAfee and McMillan (1988) provide some examples of companies showing preference for non-incumbents, though Elmaghraby (2007) discovers that this is a rare occurrence.

[2]See Froeb, Tschantz and Crooke (2001). We do offer an analytic solution to the *ex ante* symmetric case which can be viewed as the infinite horizon version of Tang Sørensen's (2006) model.

Our findings are in contrast to the related models of Bertrand and Cournot competition in the presence of customer switching costs. In these models, larger firms price less aggressively because their large base of locked-in consumers makes demand less elastic (Klemperer 1987, 1995, Farrell and Shapiro 1988, Farrell and Klemperer 2007). As a consequence, mergers that eliminate small, more aggressive, firms are considered problematic (Lofaro and Ridyard 2003). The UK Competition Commission appealed to this intuition when challenging a merger involving a bank with only a five-percent market share:

> "[I]n markets with switching costs, firms with low market share tended to grow (or sow) their share by competing aggressively ..., while those with high market share tended to exploit (or harvest) theirs by preserving or increasing margins on the existing customer base. The merger ... would replace a firm in sowing phase with one in harvesting phase, to the detriment of consumers and competition" (Lofaro and Ridyard 2003, p. 2).

In our auction setting, we find the opposite. It is the larger firms that bid more aggressively, and thus provide the greatest benefit to the auctioneer. A merger that combines two smaller bidders into one larger one will create a more aggressive bidder, and this can result in a lower price. This finding corresponds to an argument sometimes forwarded by merging parties appearing before competition agencies, that uniting two smaller firms can create a more aggressive competitor and thus promote competition. Unlike static auction merger models (e.g., Dalkir, Logan and Masson 2000, Froeb, Tschantz and Crooke 2001, Waehrer and Perry 2003), our model provides a context in which this can occur, even without cost synergies.

In what follows, we first consider bidding in markets with an incumbent advantage, which naturally gives rise to a dynamic model of competition. In the computational experiments that follow, we vary the strength of the incumbent advantage and examine the effects of mergers. In the limiting case without incumbent advantage, the model collapses to the repeated static second-price, private values auctions model. This allows us to compare the implied policy conclusions to those that follow from a static auction model. We conclude by discussing implications for competition policy and limitations of the analysis.

2 Bidding with an incumbent advantage

2.1 Model

Consider an infinite-period dynamic game between n firms, indexed by $j = 1, \ldots, n$, competing in a second-price procurement auction in each of $t = 1, 2, \ldots$ periods. We define $i^*(t)$ as the index of the incumbent bidder in period t, so that $i^*(t) = j$ if bidder j won the auction in period $t-1$ (with $i^*(1) = 0$). Every period, each firm receives an independent (though not necessarily identically distributed) cost draw, $C_j(t)$, from a time-invariant distribution $F_j(C)$ with density $f_j(C)$ and support $\mathbb{C}_j$. Costs are given by

$$c_j(t) = \begin{cases} C_j(t) & j \neq i^*(t) \\ C_j(t) - c_{inc} & j = i^*(t) \end{cases}$$

We use C to denote the elementary cost draws, and c to denote realized costs which incorporate any incumbent advantage. For all but the incumbent bidder, $c = C$, but the incumbent receives a fixed, publicly-known cost advantage of c_{inc}. This is analogous to models of auctions with synergies in which bidders receive an additional fixed value for winning all items in a desired bundle (Krishna and Rosenthal 1996, Branco 1997).

Denote by $\upsilon_j(i,t)$ firm $j's$ net present value of expected future profits at time t when i is the incumbent. Let r denote the interest rate and $p_j(i,t)$ denote firm $j's$ probability of winning the auction at time t when i is the incumbent. Define

$$\Phi_j(i^*(t),t) = \left(\upsilon_j(j,t+1) - \frac{\sum_{k \neq j} p_k(i^*(t),t)\upsilon_j(k,t+1)}{\sum_{k \neq j} p_k(i^*(t),t)} \right) \Big/ (1+r) \tag{1}$$

as the difference in future profit for bidder j between winning and losing today's auction. This is the additional value, above that implied by the profit in the current period, that a bidder places on winning and is analogous to the *intrinsic valuation* in auctions with externalities.[3] In these models, losing bidders suffer a negative externality which may depend on the identity of the winning bidder. In our case, this externality is reflected in the more preferable cost distribution for the incumbent in the subsequent period. Note that in an auction without an incumbent advantage, $\Phi_j(i^*(t),t) = 0$ and $\upsilon_j(i,t) = \upsilon_j(k,t)\ \forall i,j,k$.

We consider stationary strategies that depend only on the state variables, namely the identity of the incumbent and a firm's cost. This allows us to drop the dependence of $\upsilon_j(i^*)$, $p_j(i^*)$, and $\Phi_j(i^*)$ on time. A strategy, $b_j : \Re \times \{1,\ldots,n\} \to \Re$, maps costs into bids of player j for each possible incumbent. The Markov Perfect equilibrium bid in a second-price auction is given by:

$$b_j(c_j,i^*) = c_j - \Phi_j(i^*) \tag{2}$$

where Φ_j is the amount by which each bidder "shades" his realized cost to form bids. Note that $\Phi_j(i^*)$ depends on the winning probabilities of each firm which, in turn, depend on the bids. Denoting by $b_{-j}^{min}(i^*)$ the lowest bid of the bidders excluding j, $\{1,\ldots,j-1,j+1,\ldots,n\}$, the value function must satisfy the recursion equation:

$$\upsilon_j(i^*) = \Big(E[b_{-j}^{min}(i^*) | b_{-j}^{min}(i^*) > b_j(c_j,i^*)] - c_j \Big) p_j(i^*) + \sum_{k=1}^{n} p_k(i^*)\upsilon_j(k)/(1+r) \tag{3}$$

where the first term is the present period's profit and the second term is next period's value function for each incumbent weighted by the transition probabilities.

[3] See Jehiel, Moldovanu and Stacchetti (1996), Jehiel and Moldovanu (1996, 2000), Das Varma (2002, 2003), Bagchi and Shor (2006), and Bagchi (2008).

2.2 Logit Auction Model

To characterize bidder asymmetry in a parsimonius and tractable way, we use the logit auction model (e.g., Tschantz, Crooke and Froeb 2000, Brannman and Froeb 2000, Froeb, Tschantz and Crooke 2001). Bidder j's cost absent any incumbent advantage follows an extreme-value (Gumbel) distribution,

$$F_j(C) = 1 - e^{-e^{(C-\eta_j)/\mu}} \ for \ -\infty \leq C \leq \infty \tag{4}$$

with common variance which depends only on the spread parameter μ, but possibly different means which depend on the location parameters, η_j. This family of distributions offer two advantages for the analysis of mergers.

First, the distributions offer a natural interpretation for characterizing asymmetry between firms and for characterizing the effects of merger. The logit auction model belongs to the class of auctions characterized by independent draws from "power related families" (Waehrer and Perry 2003, Froeb, Tschantz and Crooke 2001), where bidder j receives x_j draws out of a common base distribution, $F(C)$, with his cost equaling the minimum of the draws. Bidder j's distribution is given by

$$F_j(C) = 1 - (1 - F(C))^{x_j} \tag{5}$$

Thus, a bidder taking a larger number of draws will have lower costs, on average, than those taking fewer draws (Froeb, Tschantz and Crooke 2001). If a merged firm obtains a cost equal to its constituent members' lowest-cost draw, then a merger of firms with parameters x_i and x_j has a cost distribution with parameter $x_{merged} = x_i + x_j$. This assumption has been used by enforcement agencies to characterize the effects of "naked" mergers—those without cost synergies—because merger does not change industry minimum cost (e.g., Baker 1997, Brannman and Froeb 2000, Waehrer and Perry 2003).

When the base distribution is given by $F(C) = 1 - e^{-e^{C/\mu}}$ and $x_j = e^{-\eta_j/\mu}$, firm costs are distributed extreme value (Gumbel) so that mergers reduce the mean cost draw of the merged firm, but do not otherwise change the variance. It is the only power-related family of distributions with this feature (Froeb, Tschantz and Crooke 2001). This allows us to isolate the effects of mergers without the confounding effect of a change in variance.[4]

[4]Despite their advantages, one potential pitfall of this power-related family of distributions is that they have unbounded support. This allows for both negative and arbitrary large costs. However, by choosing appropriate parameters, the probability of a negative cost draw can be made arbitrarily small. Moreover, by mixing over distributions from this family, we can approximate any joint distribution (Froeb, Tschantz and Crooke 2001).

2.3 Equilibrium

We begin with the symmetric equilibrium both because it is analytically tractable and because it serves as a reference point for the computational experiments that follow. In particular, computational experiments examine mergers to a symmetric equilibrium (post-merger symmetry) and mergers from a symmetric equilibrium (pre-merger symmetry). The symmetric equilibrium has the characteristic that increasing incumbent advantage increases the gains from winning the auction which induces more aggressive bidding on the part of all bidders.

We begin with a discussion that does not assume a specific distribution function, and then exploit the properties of the extreme value (Gumbel) distribution to derive specific results.

Let bidders receive *i.i.d.* draws from some distribution $f_j(C) \equiv f(C)$. As firms draw from identical distributions, $\upsilon_j(j) \equiv \upsilon_{inc}$ and $\upsilon_k(j) \equiv \upsilon_{non}, \ \forall j, k, \quad j \neq k$. Similarly, denote by p_{inc} and $p_{non} = (1 - p_{inc})/(n-1)$ the probability of an incumbent and non-incumbent winning the auction. Equation (1) reduces, for all j and k, to

$$\Phi_j(k) \equiv \Phi = (\upsilon_{inc} - \upsilon_{non})/(1+r) \tag{6}$$

Consider the difference between bidder j's cost and the lowest cost of rival bidders, contingent on bidder j winning. This is given by $E[\min\{c_{-j}\}| \min\{c_{-j}\} > c_j] - c_j$ where c_{-j} is the vector of costs of bidder $j's$ rivals. We denote this surplus by $\Delta^f_{inc}(c)$ when the incumbent wins with cost c and by $\Delta^f_{non}(c)$ when a non-incumbent wins. These clearly depend on the distribution, f. Denote by $\Delta^f_k \equiv E_c[\Delta^f_k(c)], k \in \{inc, non\}$, the expectations over all possible costs. These reflect the expected surpluses of the winning bidder if bids simply equalled costs. With an incumbent advantage, the winner's payment is reduced by Φ. This yields the following expressions:

$$\upsilon_{non} = (\Delta^f_{non} - \Phi)p_{non} + \frac{\upsilon_{inc}p_{non} + \upsilon_{non}(1 - p_{non})}{1+r} \tag{7}$$

$$\upsilon_{inc} = (\Delta^f_{inc} - \Phi)p_{inc} + \frac{\upsilon_{inc}p_{inc} + \upsilon_{non}(1 - p_{inc})}{1+r} \tag{8}$$

The first terms on the right hand side reflect the profit from today's auction and the second terms reflects the discounted profit of future auctions, which depend on the current auction's outcome. Solving for υ_{inc} and υ_{non} and substituting into (6), we obtain

$$\Phi = \frac{\Delta^f_{inc}p_{inc} - \Delta^f_{non}p_{non}}{1+r} \tag{9}$$

This expression indicates that each bidder bids below his costs ("shades") by the same amount, whether or not he is the incumbent, as Φ does not depend on incumbency status.

The expression for Φ provides only an implicit solution for bidding functions since p_{inc} depends on bids and the bids depend on Φ which is a function of p_{inc}. When costs are drawn from the Gumbel distribution, it is possible to derive explicit expressions for winning probabilities,

$$
\begin{aligned}
p_{inc} &= \Pr\{b_j(c_j, j) < b_i(c_i, j)\ \forall i \neq j\} = \Pr\{C_j < C_i + c_{inc}\ \forall i \neq j\} && (10)\\
&= \frac{e^{c_{inc}}}{n-1+e^{c_{inc}}} && (11)\\
p_{non} &= (1-p_{inc})/(n-1) && (12)\\
&= \frac{1}{n-1+e^{c_{inc}}} && (13)
\end{aligned}
$$

as well as equilibrium bids and expected profit,

$$
b(c) = c - \Phi = c - \log\left(1 + \frac{e^{c_{inc}}-1}{n-1}\right) \Big/ (1+r) \tag{14}
$$

$$
\Delta_k p_k = -\mu \log(1-p_k), \quad k \in \{inc, non\} \tag{15}
$$

Bids are decreasing in the incumbent advantage, c_{inc}, and equal c when the incumbent advantage is zero. That larger incumbent advantage encourages more aggressive bidding is not surprising. More interesting is the ratio of the amount of bid-shading to the incumbent advantage, $\frac{\Phi}{c_{inc}}$. This ratio equals $\frac{1}{1+r}$ when $n = 2$ suggesting that firms incorporate the entire present value of next period's cost advantage into their bid. The ratio is increasing in c_{inc}, since incumbency is easier to preserve once obtained, and decreasing in n since incumbency becomes more difficult to maintain (and is thus worth less to obtain) with more competitors.

An incumbent advantage directly reduces the equilibrium price through the incumbent's lower average cost but also indirectly through the more aggressive bidding. Thus, higher c_{inc} benefits the auctioneer, who plays the role of the consumer in this model.

As mentioned in the introduction, a symmetric setting is inappropriate for our application due to the three types of symmetry our model must accommodate: asymmetry among bidders, asymmetry created by merger, and asymmetry created by incumbent status. The cost of this modeling complexity is that we sacrifice analytic closed-form solutions in favor of a numerical solution to the asymmetric equilibrium defined by the following system of equations.

Proposition 1. *The equilibrium of the repeated second-price logit auction model with incumbent advantage, $b_j(c_j, i) = c_j - \Phi_j(i)$, is given by a system of n^2 equations in $\Phi_j(i)$,*

$$
\Phi_j(i) = \left(h_j(j) - \Phi_j(j) - \frac{\sum\limits_{k \neq j} p_k(i)\Big(h_j(k) - \Phi_j(k)\Big)}{1 - p_j(i)} \right) \Bigg/ (1+r) \tag{16}
$$

$$
\textit{where} \qquad p_j(i) = \frac{\alpha_j x_i}{\sum_{k=1}^{n} \alpha_k x_k} \qquad \alpha_k = \begin{cases} e^{\Phi_k(i)} & k \neq i \\ e^{\Phi_k(i)+c_{inc}} & k = i \end{cases} \tag{17}
$$

$$
h_j(k) = -log(1 - p_j(k)) \tag{18}
$$

The proof is in the appendix. Without symmetry, the level of bid shading, $\Phi_j(i)$, depends on the degree of incumbent advantage, c_{inc}, *and* incumbency status. However, this system of equations is transcendental, and does not lend to analytic solutions. Numerical solutions are obtained in *Mathematica* using Brent's (2002) extension to the secant method.

3 Computational Experiments

In this section, we present results of computational experiments designed to isolate the effect of an incumbent advantage on a merger. We hold the underlying cost distributions of the bidders fixed and let $\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n)$ be a vector of cost parameters, $x_j > 0$. A merger among firms 1 and 2 creates an industry with $n-1$ bidders with cost parameters $(x_1 + x_2, x_3, \ldots, x_n)$. We set the spread parameter, $\mu = 1$, and for ease of interpretation, normalize $\sum x_j = 1$, so that x_j is precisely firm j's probability of winning without incumbent advantage. Lastly, the interest rate, r, is set to 0.1. Varying the spread parameter or interest rate does not qualitatively change the results. Intuitively, increasing the interest rate moves the model predictions closer to the predictions from the static (non-repeated) model.

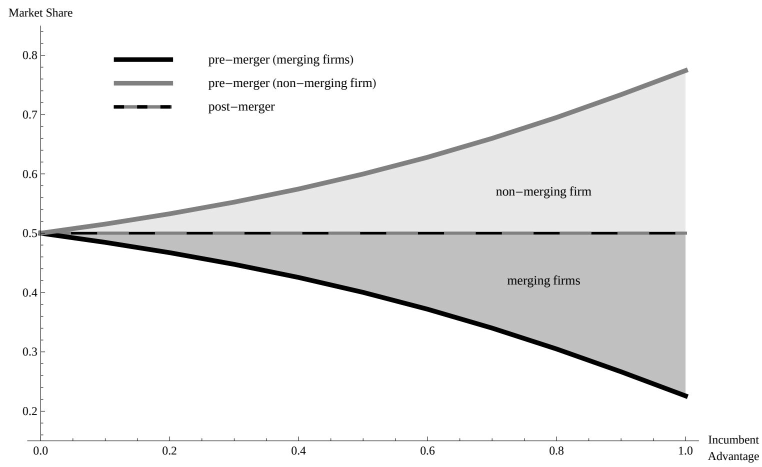
We present three sets of computational experiments. First is a merger to symmetry in a three-firm industry, so that post-merger industry is characterized by two symmetric firms. Second is a merger from symmetry in a three-firm industry where pre-merger, all three firms are identical. Lastly, we consider arbitrary mergers of any two firms in both 3-firm and 6-firm industries. This allows us to examine the interplay between incumbent advantage, industry size, and the concentration of the non-merging fringe.

3.1 Mergers to symmetry

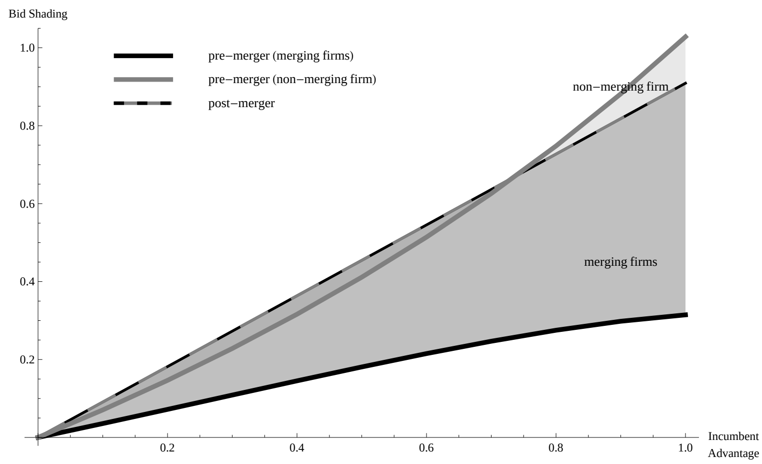
In this section, we compute the effects of moving from an industry characterized by $\mathbf{x} = (1/4, 1/4, 1/2)$ to a post-merger industry characterized by $\mathbf{x} = (1/2, 1/2)$. Thus, pre-merger, firms one and two have less favorable cost distributions. Following a merger of these two firms, the industry becomes symmetric. In the computational experiments that follow, we reach three conclusions about the effects of mergers in the presence of an incumbent advantage:

- mergers increases the combined share of the merging firms,
- mergers increases bidding aggressiveness by the merging firms, and
- mergers are always profitable and often reduce price.

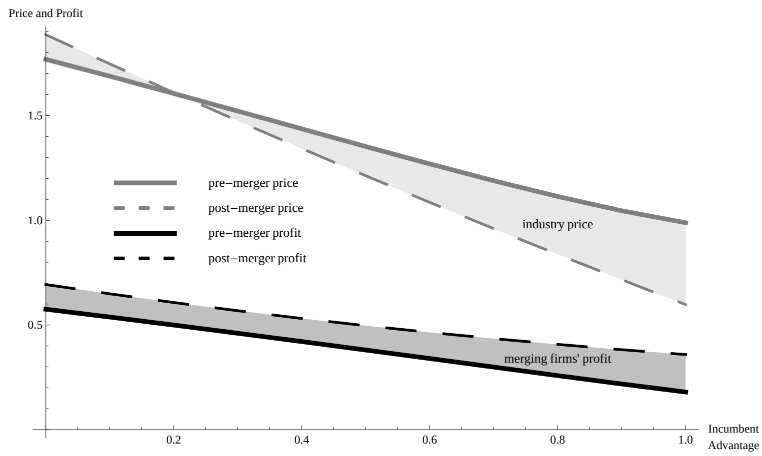
Figure 1 illustrates the merger effects as the incumbent advantage c_{inc} varies. In the top panel, we illustrate the effect of the merger on market shares. Pre-merger (solid line), the merging firms' total market share (black line) is the mirror image of the non-merging firm's share (gray line). As incumbent advantage increases, the market share of the larger non-merging firm also



(a) Shares: merging firms become larger



(b) Bid Shading: merging firms bid more aggressively



(c) Prices and Profit: mergers are profitable and can reduce industry price

Figure 1: **Merger to Symmetry**: from a pre-merger cost distribution of (1/4, 1/4, 1/2) to a post-merger distribution of (1/2, 1/2). In each of the panels, the solid line represents the pre-merger equilibrium, and the dotted line, the post-merger equilibrium. In top panel of this figure, the merging firm *gains* market share; in the middle panel, the merging firm bids much more aggressively post-merger, explaining its post-merger gain in share; and in the bottom panel, mergers are always profitable, and can reduce expected post-merger price.

increases. Post-merger (dashed line), the merging and non-merging shares are equal at 50%. When incumbent advantage equals zero, the sum of the shares of the pre-merger firms equals the share of the post-merger firm and mergers have no effect on shares. This is because the non-merging firm's probability of having the lowest cost draw does not change with the merger of its rivals. With incumbent advantage, since the merger always results in symmetric market shares, the merger has the effect of increasing the share of the merging firm, while decreasing the share of the non-merging firm.

In the middle panel, we plot the average level of bid shading below one's costs of the merging and non-merging bidders. Since the level of bid-shading depends on whether one is the incumbent, the figure displays the average level of bid shading ($\Phi_j(i)$ weighed by the probability of incumbency for i). The merging firm bids much more aggressively following the merger, while bidding behavior of the non-merging firm remains largely unchanged. The more aggressive bidding by the merged firm causes its share to increase in the top panel.

In the bottom panel, we consider the incentives to merge and the merger's effect on price. For low levels of incumbent advantage, the merger is anticompetitive. This corresponds to the predictions of the static auction model where the expected price increase is the probability that the merging firms draw the two-lowest costs times the difference between the second- and the third-lowest cost. In essence, the merged firm bids less aggressively, and this raises price.

However, as incumbent advantage increases, the merged firm bids more aggressively. Whether this leads to a lower price depends also on the bids of the non-merging bidder and how often the merged firm wins and how often it loses and sets the price. Symmetry-creating mergers in the presence of a larger incumbent advantage are likely to be pro-competitive. In addition, we see that these mergers are profitable over the entire range of incumbent advantage, even when the merger causes expected price to fall.

3.2 Mergers from symmetry

In this section, we consider an industry that is symmetric prior to merger. A three-to-two merger moves an industry with $\mathbf{x} = (1/3, 1/3, 1/3)$ to a post-merger industry with $\mathbf{x} = (2/3, 1/3)$. The merged firm has the most favorable cost distribution following the merger. In the computational experiments that follow, we reach three conclusions about the effects of mergers in the presence of incumbent advantage. Note that the first two conclusions are the same as for mergers from symmetry, but the third is more qualified, i.e. mergers "sometimes" reduce price instead of "often" reduce price.

- mergers increase the combined share of the merging firms,
- mergers increase bidding aggressiveness by the merging firms, and
- mergers are always profitable and sometimes reduce price.

Figure 2 illustrates the merger effects. In the top panel, the pre-merger industry is symmetric, so the merging firms have combined twice the market share of the non-merging firm and these shares do not vary with incumbent advantage. Post-merger, however, the merged firm's market share (black) grows with incumbent advantage. As in the previous section, increasing the incumbent advantage exacerbates cost asymmetries, increasing the share of the lower-cost, merged firm.

The middle panel shows that the merging and non-merging firms shade their bids equally in the pre-merger symmetric world. It is only in the post-merger world, as incumbent advantage grows, that the lower-cost merging firm bids more aggressively, accounting for the share increase in the top panel. The non-merging firm also bids slightly more aggressively post-merger for low levels of incumbent advantage. When the incumbent advantage is small, the merger causes both firms to bid more aggressively. For larger incumbent advantage, the non-merging firm begins to bid less aggressively, effectively ceding share to the lower-cost merged rival.

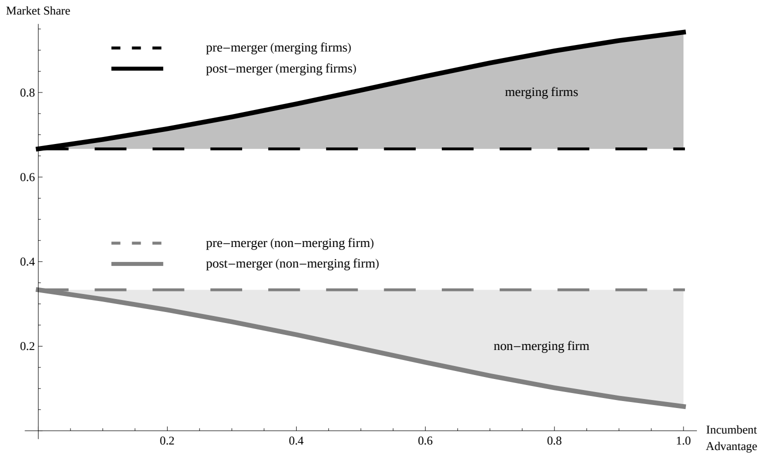
The bottom panel reflects both the incentive to merge and the anti-competitive effects of mergers. This three-to-two merger from symmetry is profitable over the entire range of incumbent advantage and always causes price to increase. However, the more aggressive bidding by both bidders for low levels of incumbent advantage suggests that the price effects of a merger are first decreasing than increasing in incumbent advantage (lightly shaded area in panel c). Thus, in a three-to-two merger from symmetry, static models that ignore incumbent advantage will accurately predict the direction of price movements, but may overestimate or underestimate their magnitude. This error is more pronounced in industries with more than three firms.

3.3 Non-symmetric mergers

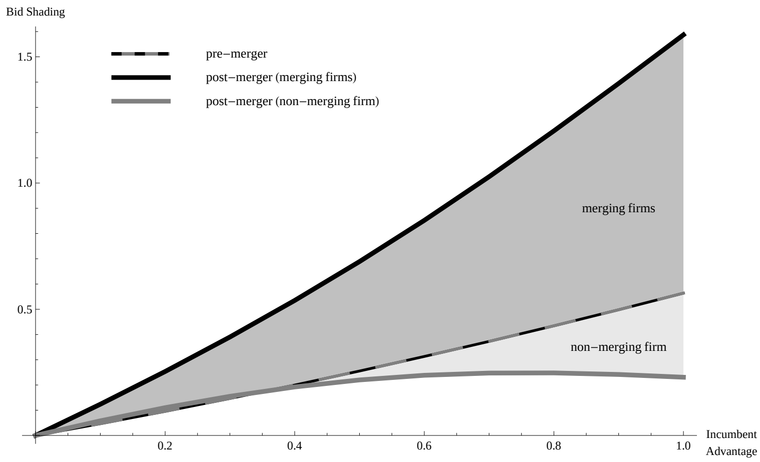
In the preceding subsections, we considered the polar cases of industries that are symmetric either before or after the merger. In both cases, we assumed that merging firms were of equal size. In this section, we examine mergers among firms of differing sizes. We also consider a less concentrated non-merging fringe. Figure 3 shows the price effects of a merger between two firms for different market sizes and levels of the incumbent advantage. In mergers from three to two firms (top two panels), mergers among small firms (lower left of each panel) are pro-competitive even under fairly small levels of incumbent advantage. In a six-firm industry, mergers among small firms are unlikely to be pro-competitive when the incumbent advantage is small. Several insights may be gleaned from this figure.

First, the thick isoprice line (which indicates no merger price effect) is nearly linear, suggesting that the sum of merging firms' cost parameters may be a better indicator of merger effects than the sum of the products of their shares, as in the HHI.

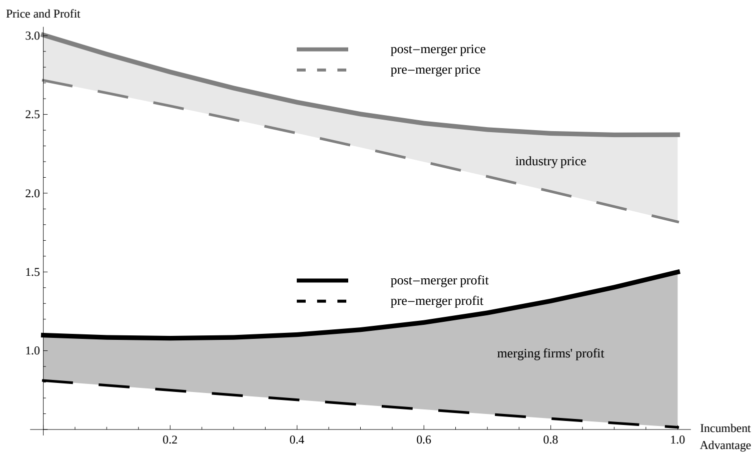
Second, a merger of two firms with given cost parameters is more likely to generate pro-competitive effects in markets with few competitors than in markets with many competitors. This means that the order in which mergers occur determines whether they are anticompetitive



(a) Shares

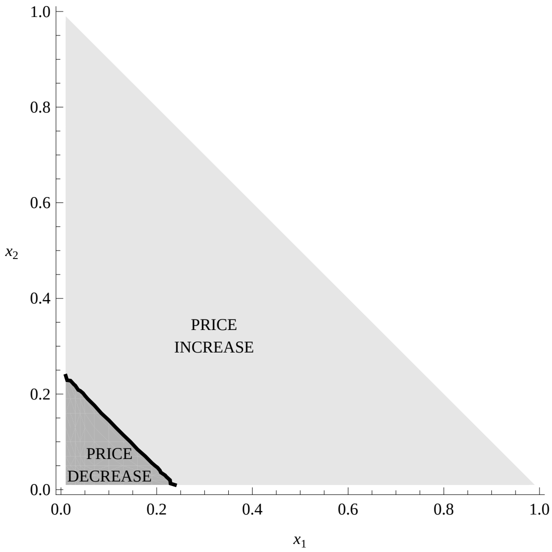


(b) Bid Shading

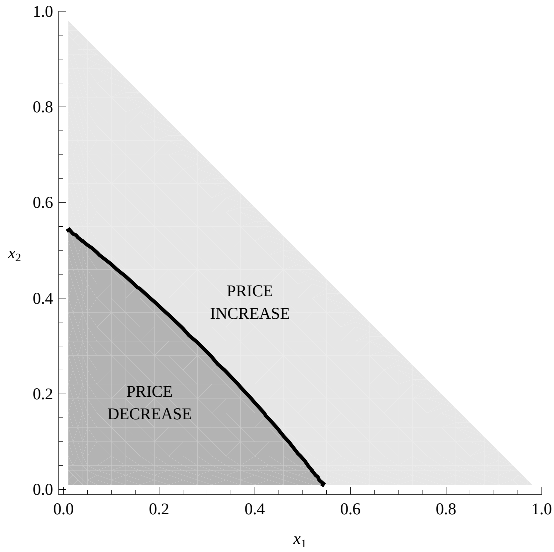


(c) Prices and Profit

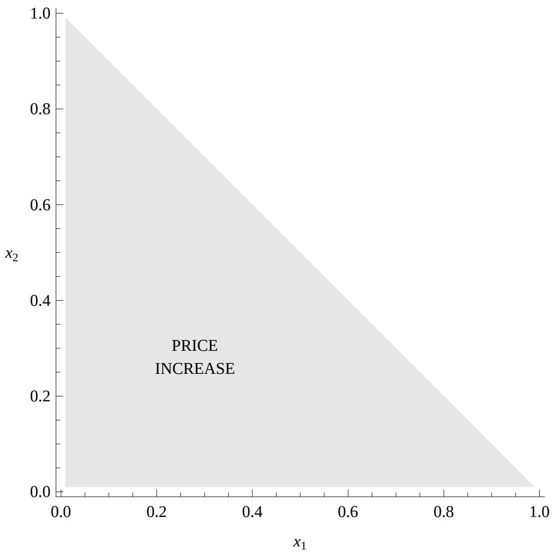
Figure 2: Merger From Symmetry: from a pre-merger cost distribution of (1/3, 1/3, 1/3) to a post-merger cost distribution of (2/3, 1/3). In top panel of this figure, the merging firm *gains* market share; in the middle panel, the merging firm bids much more aggressively post-merger while the non-merging firm bids less aggressively, explaining its share increase; and in the bottom panel, mergers are always profitable, and post-merger price always increases.



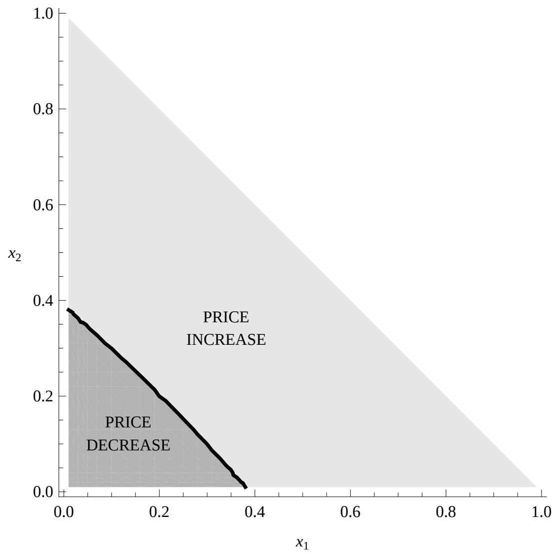
(a) three to two merger, small incumbent advantage



(b) three to two merger, large incumbent advantage



(c) six to five merger, small incumbent advantage



(d) six to five merger, large incumbent advantage

Figure 3: **Incumbent advantage increases the scope for pro-competitive mergers**. Panels show price effects of a merger among two firms with cost parameters x_1 and x_2 under small ($c_{inc} = 0.05$) and big ($c_{inc} = .5$) incumbent advantage. Lightly shaded region shows an (anticompetitive) increase in price, while darker region illustrates prices declining as a result of merger. With three firms, non-merging firm has cost parameter $1 - x_1 - x_2$; with six firms, each non-merging firm has cost parameter $(1 - x_1 - x_2)/4$. Prices are more likely to decline as a result of merger when merging firms are small, the market has few firms, and incumbent advantage is large.

or not. Consider for example a six-firm industry with an incumbent advantage of $c_{inc} = .5$ and cost parameters of $x_1 = x_2 = 0.28$ and $x_3 = x_4 = x_5 = x_6 = 0.11$. A merger between firms 1 and 2 would increase price (panel d). However, if the four smaller firms merge first, then a merger between firms 1 and 2 would actually be pro-competitive (panel b). Contrary to the traditional thinking that merger effects grow with increasing consolidation, it is the early mergers in this example that harm the industry, and the latter merger that increases competition.

Third, the above example also demonstrates that even some mergers that create a dominant firm may be pro-competitive. The three-to-two merger of firms 1 and 2 creates a firm with cost parameter 0.56. Pre-merger, these firms were too small to compete effectively against their larger, often incumbent-advantaged, rival. Post-merger, their more aggressive bidding reduces average industry price.

4 Policy Implications

The difficulty of translating these theoretical results into policy guidance is that observed firm shares do not directly translate into the unobserved parameters of cost distributions, as the link is conflated with incumbent advantage. To illustrate, consider an industry with three firms, two of which each have 10% market share. Without incumbent advantage, a firm with 10% market share in a second-price auction has precisely a 10% chance of having the lowest cost. With incumbent advantage, asymmetries in the cost distribution are reflected in even larger asymmetries in observed shares. Firms with 10% market share might have a 25% chance of having the lowest cost but the more aggressive bidding by their larger rival reduces their probability of winning to 10%. If these two smaller firms merge, the resulting industry would be symmetric, and their shares would increase from a combined 20% to 50%. A simple analysis that merely adds the shares of the two smaller firms to determine the post-merger share will, of course, conclude that the post-merger HHI is higher than pre-merger. This is only part of the story. The symmetry-inducing merger actually leads to a decrease in the post-merger HHI and a decrease in industry price. Pre-merger shares do not accurately characterize the effects of a merger without knowledge of the incumbent advantage.

One way to think about the difficulty is to recognize that firms have not one, but two "shares," or winning probabilities, depending on whether they are the incumbent. If incumbent advantage is strong, then the probability of winning an auction conditional on being the incumbent is much larger than the probability of winning conditional on being the non-incumbent. If the incumbent advantage is weak then the two probabilities are much closer together. With no incumbent advantage, corresponding to a static model, the two probabilities are equal.

In Figure 4, we plot the region where two mergers reduce price in the space of these two probabilities. On the horizontal axis is the probability of winning by a non-incumbent and on the vertical axis is the probability of winning by the incumbent. The 45 degree line, where

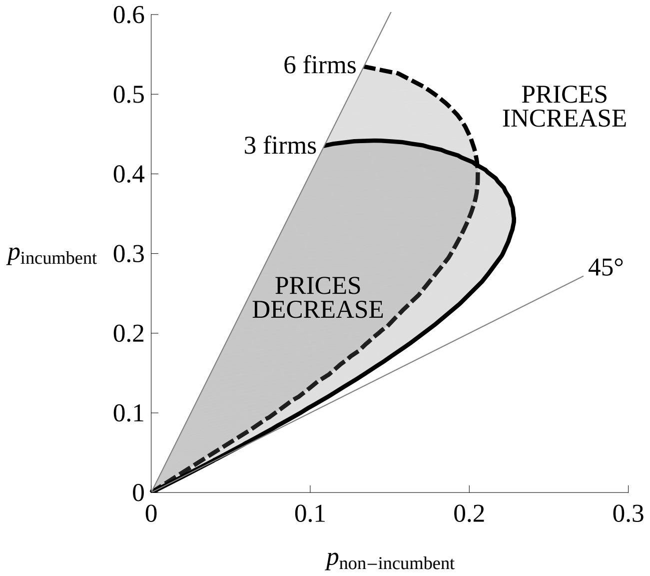


Figure 4: **Pro-competitive mergers are more likely the more that incumbency increases the probability of winning**: *Probability of incumbent win* vs. *probability of non-incumbent win*. This graph plots the region where mergers decrease expected industry price. The three-firm industry has shares of $(x, x, 1-2x)$ and the six-firm industry has shares of $(x, x, (1-2x)/4, (1-2x)/4, (1-2x)/4, (1-2x)/4)$. We plot the region in the space of the pre-merger probabilities of winning conditional on being the incumbent (vertical axis) and conditional on being a non-incumbent (horizontal axis) for one of the merging firms.

the two probabilities are equal, corresponds to the static merger model, associated with no incumbent advantage. The shaded regions correspond to the the areas where mergers in two different industries are pro-competitive, i.e., post merger industry price decreases. The region with the border labeled "3 firms" corresponds to an industry with cost parameters $(x, x, 1-2x)$ and the region labeled "6 firms" corresponds to an an industry with $(x, x, (1-2x)/4, (1-2x)/4, (1-2x)/4, (1-2x)/4)$. To generate the regions, we vary x and c_{inc}, the degree of incumbent advantage, and mark the areas where industry price decreases.

Along the 45 degree line, there is no incumbent advantage and mergers always raise price as in the static model. Similarly, mergers are anticompetitive when the merging firms are large (up and to the right). In this region, bidding becomes less aggressive because larger firms are harvesting incumbency status and smaller firms are bidding as if there is little to gain by becoming the incumbent. Only when the incumbent advantage becomes large (above the 45 degree line) is there a possibility of a pro-competitive merger. Where incumbency advantage is larger, and the merging firms smaller, we see the effects we previously identified in our computational experiments. Summarizing, pro-competitive mergers are more likely:

- the larger the incumbent advantage,
- the smaller the shares of the merging firms firms, and
- the more concentrated is the non-merging fringe.

5 Conclusion

Farrell and Klemperer (2007) introduce their chapter on switching costs in the *Handbook of Industrial Organization* by warning that the analysis of mergers is different in the presence of switching costs. In an auction instead of a price-setting environment and with an incumbent advantage instead of switching costs, we reach the same conclusion. We find that competitive behavior in repeated auctions with an incumbent advantage is characterized by dynamic concerns analogous to those that characterize competitive behavior in the presence of switching costs. Firms' bids reflect a similar inter-temporal tradeoff and mergers affect the trade-off for merging and non-merging firms alike. As a consequence, the welfare effects of mergers depend critically on the degree of incumbent advantage. However, rather than justifying "increased antitrust scrutiny" (p. 2005), we find that the competition in this environment gives rise to a greater scope for pro-competitive mergers.

This conclusion may not be very useful for distinguishing pro-competitive from anti-competitive mergers because the relationship between unobserved costs and observed shares is confounded by the magnitude of incumbent advantage. Determining how to empirically identify the cost parameters from the share parameters would seem like a worthwhile econometric exercise. One obvious identification strategy is to examine the difference between win rates for incumbent and non-incumbent firms. Without knowing whether a firm's large winning probability is due to increased incumbent advantage or an inherent product or cost advantage, both type I (over-deterrence) and type II (under-deterrence) enforcement errors are possible.

We model incumbent advantage as short-lived, lasting only as long as a firm remains an incumbent and disappearing as soon as a firm loses. This is an accurate description of cases where sunk costs must be incurred with each new contract. If incumbency leads to long-lived advantages, such as learning curves which decay over time, we speculate that similar results will likely obtain. The current model where a current state is defined by the index of the current incumbent would instead require a definition of a state as a vector of each firm's current stock of learning. Further, the value of next period's incumbency would replace a single-period benefit with a discounted future stream of benefits. Yet, the main intuition that a firm with an *ex ante* advantage would have a greater chance of retaining (and thus a greater value of obtaining) incumbency would likely survive.

Our conclusions are limited, as always, to our specific setting. In particular, we consider only myopic consumers (auctioneers) in second-price, private-value settings, and firms that cannot commit to future prices. In this setting, the presence of incumbent advantage leads to substantially different predictions about the competitive effects of mergers than those obtained in the absence of incumbent advantage.

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Appendix

Proof of Proposition 1. The probability of bidder j winning given that i^* is the incumbent is given by:

$$p_j(i^*) = \frac{\alpha_j x_i}{\sum_{k=1}^n \alpha_k x_k} \qquad \text{where } \alpha_k = \begin{cases} e^{\Phi_k(i^*)} & k \neq i^* \\ e^{\Phi_k(i^*)+c_{inc}} & k = i^* \end{cases} \tag{19}$$

Substituting into (1), and dropping the dependence on time, provides

$$\Phi_j(i^*) = \left(\upsilon_j(j) - \frac{\sum_{k\neq j} p_k(i^*)\upsilon_j(k)}{\sum_{k\neq j} p_k(i^*)}\right)/(1+r) \tag{20}$$

$$= \frac{\upsilon_j(j) - \sum_{k=1}^n p_k(i^*)\upsilon_j(k)}{(1+r)(1-p_j(i^*))} \tag{21}$$

The expected surplus in the present period is given by:

$$\left(E[b_{-j}^{min}(i^*)|b_{-j}^{min}(i^*) > b_j(c_j, i^*)] - b_j(c_j, i^*)\right) p_j(i^*) = -\log(1-p(j,i^*)) - \Phi_j(i^*)p_j(i^*) \tag{22}$$

Substituting into (3), we obtain

$$\upsilon_j(i^*) = -\log(1-p_j(i^*)) - \Phi_j(i^*)p_j(i^*) + \sum_{k=1}^n p_k(i^*)\upsilon_j(k)/(1+r) \tag{23}$$

$$= -\log(1-p_j(i^*)) - \Phi_j(i^*)p_j(i^*) - (1-p_j(i^*))\Phi_j(i^*) + \upsilon_j(j)/(1+r) \tag{24}$$

$$= -\log(1-p_j(i^*)) - \Phi_j(i^*) + \upsilon_j(j)/(1+r) \tag{25}$$

and in particular

$$\upsilon_j(j) = -\log(1-p_j(j)) - \Phi_j(j) + \upsilon_j(j)/(1+r) \tag{26}$$

$$= -\left(\log(1-p_j(j)) + \Phi_j(j)\right)\frac{1+r}{r} \tag{27}$$

and therefore

$$\upsilon_j(i^*) = -\log(1-p_j(i^*)) - \Phi_j(i^*) - \left(\log(1-p_j(j)) + \Phi_j(j)\right)/r \tag{28}$$

Finally, substituting the value of $\upsilon_j(i^*)$ into (20),

$$\Phi_j(i^*) = \frac{\upsilon_j(j) - \sum_{k=1}^n p_k(i^*)\upsilon_j(k)}{(1+r)(1-p_j(i^*))} \tag{29}$$

$$= \frac{-\log(1-p_j(j)) - \Phi_j(j) - \sum_{k=1}^n p_k(i^*)\Big(-\log(1-p_j(k)) - \Phi_j(k)\Big)}{(1+r)(1-p_j(i^*))} \qquad \square$$