Rev. Econ. Design (2013) 17:183–203
DOI 10.1007/s10058-013-0143-0

ORIGINAL PAPER

Information concentration in common value environments

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Received: 1 April 2009 / Accepted: 16 April 2013 / Published online: 8 May 2013

Abstract We consider how information concentration affects a seller's revenue in common value auctions. The common value is a function of n random variables partitioned among $m \leq n$ bidders. For each partition, the seller devises an optimal mechanism. We show that whenever the value function allows scalar sufficient statistics for each player's signals, the mechanism design problem is well-defined. Additionally, whenever a common regularity condition is satisfied, a coarser partition always reduces revenues. In particular, *any* merger or collusion among bidders reduces revenue.

Keywords Information concentration · Industry concentration · Mechanism design · Common value auctions

JEL Classification D44 · L41 · C72 · D82

1 Introduction

Consider a seller of an indivisible item facing several potential buyers, each with some information about the object's common value. How would a decision by a group

The authors would like to thank Luciano de Castro, Luke Froeb, Paul Klemperer, Rich McLean, Rakesh Vohra, and an anonymous referee for valuable comments and suggestions.

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of buyers to centralize their information and decision-making authority impact the seller's revenue? While the question is of obvious interest to auctioneers considering allowing bidding syndicates and to policy governing mergers and collusion, these environments remain largely unmodeled. Providing a general result on the revenue impact of information concentration is the purpose of this manuscript.

A growing literature is devoted to determining how access to more informative signals changes behavior in allocation problems (Matthews 1984; Persico 2000; Athey and Levin 2001; Bergemann and Valimaki 2002; Mares and Harstad 2003). Since these approaches generally define "more informative" in terms of some order over scalar random variables, they permit two buyers working together to possess anything from only a slightly more informative signal than each of them had individually to nearly perfect information about the object's value. Another strand of literature finds that a more concentrated industry, obtained by removing a bidder *and his information*, is less profitable even when the auctioneer reacts with an optimal mechanism (Bulow and Klemperer 1996). However, this approach conflates the role of industry concentration with information concentration.

In contrast, our approach keeps the total amount of information constant while concentrating its allocation among fewer bidders. The centralization of two buyers' signals would simply have the joint entity with two signals. Since the new entity possesses a multidimensional signal, this calls into question the existence of equilibria in auctions (Jackson 2009) and of incentive-compatible mechanisms in general (Armstrong and Rochet 1999). Several authors have adopted models of bidding with multidimensional signals while imposing symmetry (Goeree and Offerman 2002; DeBrock and Smith 1983; Mares and Shor 2008). Yet, symmetric models are particularly ill-suited to modeling mergers or collusion as even an *a priori* symmetric industry will not be so following a merger. [1] Without symmetry, simple auction mechanisms need not be optimal. In this paper, we provide the auctioneer full strategic latitude in the choice of mechanisms. In private value auctions, Waehrer and Perry (2003) find that merger effects can be partially offset by strategically altering the reserve price.

In our model, the object's value is a function of n independent (but not necessarily identically distributed) signals, which are allocated among $m \leq n$ bidders. We adopt a mechanism design approach, allowing the seller to select an optimal mechanism for each allocation of signals among bidders. We require that each bidder's vector of signals allows a scalar sufficient statistic, a condition satisfied by commonly analyzed models including additively separable and maximum or minimum value auctions (Mares and Shor 2008; Bikhchandani and Riley 1991; Bulow and Klemperer 2002; Krishna and Morgan 1997). Our central result is that a coarser partition of information among bidders always results in reduced revenue for the seller. This implies that *all* mergers reduce the auctioneer's revenue, even those that make an industry more symmetric by aggregating smaller bidders.

[1] Common value auctions appear especially sensitive to asymmetries. Bikhchandani and Riley (1991) notes that even vanishing asymmetries can lead an advantaged bidder to win the auction with probability 1. Klemperer (1998) provides discussion of this result in relation to the FCC spectrum auctions. Symmetric models can significantly overstate price changes following a merger of smaller firms in asymmetric industries (Dalkir et al. 2000; Tschantz et al. 2000).

To approach the mechanism design problem in asymmetric, multidimensional signals contexts, we provide a result that significantly simplifies the problem. Whenever a scalar sufficient summary statistic exists for each player's signals, every incentive-compatible mechanism has a revenue-equivalent incentive-compatible *scalar mechanism* that requires only scalar reports. We offer a constructive proof of this result in a general environment. This allows a broad class of problems to be analyzed in the Myerson (1981) framework and provides sufficient conditions for the existence of incentive-compatible mechanisms.

For example, Biais et al. (2000) and Goeree and Offerman (2002) consider specific value functions for which two signals can be summarized with a scalar sufficient statistic. DeBrock and Smith (1983) employ conditional log-normal distributions of signals and use the geometric mean as a sufficient statistic of value. While the existence of scalar sufficient statistics is not a general property, it is reasonably broad to produce an interesting class of analytical examples. To illustrate, consider quasi-arithmetic means,

$$M_f(\mathbf{x}) = f^{-1}\left(\frac{\sum f(x_i)}{n}\right),$$

for some increasing f and define the value function, $V(\mathbf{x}) = g\left(M_f(\mathbf{x})\right)$, as a function of signals $\mathbf{x}$ for some increasing g. When $f(x) = x^\alpha$, we obtain the class of power means $M_{x^\alpha}(\mathbf{x}) = \left(\sum \frac{1}{n} x_i^\alpha\right)^{1/\alpha}$ which encompasses the additive model and the interesting limit cases $\lim_{\alpha\to\infty} M_{x^\alpha}(\mathbf{x}) = \max(\mathbf{x})$, $\lim_{\alpha\to 0} M_{x^\alpha}(\mathbf{x}) = \prod x_i^{1/n}$, and $\lim_{\alpha\to -\infty} M_{x^\alpha}(\mathbf{x}) = \min(\mathbf{x})$. The minimum and maximum value functions have been studied by Mares and Harstad (2003) while mergers in the context of a *symmetric* additive model have been analyzed by Krishna and Morgan (1997) and Mares and Shor (2008). It is obvious that $M_f(\mathbf{s})$ is a scalar sufficient statistic for any $\mathbf{s}$, even for nonsymmetric value functions.

It is also worth mentioning that while the existence of scalar sufficient statistics is broadly a sufficient condition for the existence of equilibria in certain auctions, it is by no means a necessary condition. However, generically, one can construct examples where the absence of scalar representations induces non-monotonicity of or even absence of equilibria (Jackson 2009; Reny and Zamir 2004).

In what follows, we describe a model of common value environments that allows for arbitrary partitions of signals among bidders. We first present our result on the existence of optimal scalar mechanisms. This allows us to extend the definition of virtual valuations to these contexts. Then, we present our main result that coarser partitions of information in regular allocation problems decreases revenue. We demonstrate these results for a class of value functions and conclude with policy implications and suggestions for future research.

2 Model

A seller has one indivisible object. Consider a collection of n independent random variables, $(X_1, \ldots, X_n)$, where X_i has density f_i, distribution F_i, survival function

$\bar{F}_i$, and typical element $x_i \in [\underline{z}_i, \overline{z}_i] \subset \Re$. These random variables need not be identically distributed, but we do require independence. Without independence, the results of Crémer and McLean (1988) and McAfee and Reny (1992) would imply full revenue extraction, trivially eliminating any impacts information concentration may have on revenue as long as two buyers remain.

An *information profile* $\mathbf{A} = \{A_1, \ldots, A_m\}$ is a partition of the index set $\{1, \ldots, n\}$, with the interpretation that the n signals are distributed among $m \leq n$ risk-neutral buyers. $\mathbf{S}_i \equiv \{X_k\}_{k \in A_i}$ describes bidder i's private information with typical element $s_i \in \Delta_i \equiv \prod_{k \in A_i} [\underline{z}_k, \overline{z}_k] \subset \Re^{|A_i|}$. A vector of realizations of private information is denoted $\mathbf{s} = (\mathbf{s}_i, \mathbf{s}_{-i}) = (\mathbf{s}_1, \ldots, \mathbf{s}_m)$.

Bidder i's valuation for the object is given by $V_i(\mathbf{s})$ which is increasing in $\mathbf{s}_i$ and non-decreasing in $\mathbf{s}_{-i}$. This model generalizes the standard symmetric auction model. The special case of each bidder possessing one identically distributed signal is obtained when $m = n$, $A_i = \{i\}$, and $F_i \equiv F$.

The seller constructs a mechanism, $\eta = (p_i(\mathbf{s}), \xi_i(\mathbf{s}))$, which determines allocation probabilities and payments based on buyers' reports of $\mathbf{s}$. Define an information profile $\mathbf{A}'$ as *coarser* than information profile $\mathbf{A}$ if, for each $A \in \mathbf{A}$, there exists an $A' \in \mathbf{A}'$ such that $A \subseteq A'$. For example, consider a merger between bidders 1 and 2, so that the merged entity shares all information and places a single bid. By redistributing all of bidder 2's information toward 1, the post-merger information profile, $\mathbf{A}' = \{A_1 \cup A_2, A_3, \ldots, A_m\}$, is coarser than the pre-merger profile, $\mathbf{A} = \{A_1, A_2, A_3, \ldots, A_m\}$.

3 Scalar mechanisms

We will show that when each player's type is multidimensional but admits a sufficient statistic, the search for optimal mechanisms is simplified. *Scalar mechanisms*, which require only the report of the sufficient statistic instead of the full vector describing the private information, are natural candidates in such settings. Their reliance on scalar private reports for the allocation, and payment problem reduces the dimensionality of the message space under consideration. We will show that they can replicate the revenue properties of their more complex counterparts.

Assume that, for information profile $\mathbf{A} = \{A_1, \ldots, A_m\}$ and for player i, the value functions admit sufficient statistic representations. Formally, there exists a function $\phi^{A_i} : \Delta_i \to \Re$ which satisfies, for all j and any $\mathbf{s}_{-i}$,

$$V_j(\mathbf{s}_i, \mathbf{s}_{-i}) \geq V_j(\mathbf{s}'_i, \mathbf{s}_{-i}) \Leftrightarrow \phi^{A_i}(\mathbf{s}_i) \geq \phi^{A_i}(\mathbf{s}'_i).$$

We define two types $\mathbf{s}_i$ and $\mathbf{s}'_i$ as *equivalent* if $\phi^{A_i}(\mathbf{s}_i) = \phi^{A_i}(\mathbf{s}'_i)$.

For every mechanism, $\eta = (p_i(\mathbf{s}_i, \mathbf{s}_{-i}), \xi_i(\mathbf{s}_i, \mathbf{s}_{-i}))$ we can construct a *scalar mechanism* $\eta' = (p'_i(\mathbf{s}_i, \mathbf{s}_{-i}), \xi'_i(\mathbf{s}_i, \mathbf{s}_{-i}))$ where

$$p'_j(\phi^{A_i}(\mathbf{s}_i), \mathbf{s}_{-i}) = \int p_j(\mathbf{t}_i, \mathbf{s}_{-i}) f_i\left(\mathbf{t}_i | \phi^{A_i}(\mathbf{t}_i) = \phi^{A_i}(\mathbf{s}_i)\right) d\mathbf{t}_i, \quad \text{and} \tag{1}$$

$$\xi'_j(\phi^{A_i}(\mathbf{s}_i), \mathbf{s}_{-i}) = \int \xi_j(\mathbf{t}_i, \mathbf{s}_{-i}) f_i\left(\mathbf{t}_i | \phi^{A_i}(\mathbf{t}_i) = \phi^{A_i}(\mathbf{s}_i)\right) d\mathbf{t}_i. \tag{2}$$

for every j and $\mathbf{s}_i$. The scalar mechanism, η', averages the allocation probabilities and payment functions over the set of equivalent types. Two equivalent types $\mathbf{s}_i$ and $\mathbf{s}'_i$ will receive the asset with the same probability $p'_i(\phi^{A_i}(\mathbf{s}_i), \mathbf{s}_{-i})$ and face the same payment $\xi'_i(\phi^{A_i}(\mathbf{s}_i), \mathbf{s}_{-i})$. Furthermore, from player j's perspective, the allocation probability and payment are invariant across equivalent types of player i. In essence, the mechanism η' requires only a report of the sufficient statistic of player i's private information.

We show that mechanisms η and η' are revenue equivalent. Additionally, if η is incentive compatible, then so is η', so any mechanism has an associated revenue-equivalent scalar mechanism.

Theorem 1 *For any set of value functions which admit scalar sufficient statistics for player i, and any incentive-compatible mechanism η, we can construct an incentive-compatible scalar mechanism, η', which has the same expected revenue as η but which has probability and payment functions which depend only on the sufficient statistic ϕ^{A_i} of the private information of player i.*

The theorem addresses a single bidder whose information can be summarized via a sufficient statistic. Its extension to instances where multiple or even all players' private information can be captured by scalars is straightforward, applying the theorem to each player in turn. Scalar mechanisms are easier to implement than mechanisms requiring multiple reports from each bidder, especially when different bidders are asked for different quantities of reports.

The theorem guarantees that a scalar mechanism exists in the set of all optimal mechanisms for this problem. This allows the search for optimal mechanisms to consider only scalar mechanisms. The next section exploits this property by establishing, for a certain class of value functions, a strong order among revenues generated by different information partitions.

4 Information concentration

We now investigate the impact of information concentration on the seller's revenues. We will treat information concentration as a redistribution of the available information among bidders. Intuitively, one can think of a process of coalition formation whereby bidders " buy out" their competitors, gaining access to their private information. Each such merger among bidders transforms the information profile, $\mathbf{A}$, into a coarser information profile, $\mathbf{A}'$.

For the remainder of the manuscript, we concentrate on pure common value auctions, $V_i(\cdot) \equiv V(\cdot)$, where the value of the object is the same for each bidder under every information profile. This implies that mergers do not have value-improving synergies and allows us to focus on the role of information concentration. This also allows us to sidestep inherent problems of ad hoc value formulations in post-merger scenarios. We place the following assumptions on V:

Assumption 1 V is twice differentiable, increasing, and weakly supermodular: $\partial_i V > 0$ and $\partial_{ij} V \geq 0$ for all i, j.

Assumption 2 V admits sufficient statistic representations for all players and information profiles $\mathbf{A} = \{A_1, \ldots, A_m\}$ such that ϕ^{A_i} is differentiable in all its arguments, for all i.[2]

Since sufficient statistics are uniquely defined only up to a monotone transformation, we arbitrarily select one set of representations. It will be convenient to refer to value functions parametrized over the space of sufficient statistics, rather than the elementary signals. Define implicitly $V^{A_i} : \Re^{1+\sum_{j\neq i}|A_j|} \to \Re$

$$V^{A_i}(t_i, \mathbf{s}_{-i}) = V(\mathbf{s}_i, \mathbf{s}_{-i})$$

whenever $\phi^{A_i}(\mathbf{s}_i) = t_i$. This is the value function when bidder i's (possibly multidimensional) information is replaced with its scalar sufficient statistic. Similarly, define for every information profile $\mathbf{A} = \{A_1, \ldots, A_m\}$, the function $V^{\mathbf{A}} : \Re^{|\mathbf{A}|} \to \Re$

$$V^{\mathbf{A}}(t_1, \ldots, t_m) = V(\mathbf{s}_1, \ldots, \mathbf{s}_m)$$

if, for all $i \in \{1, \ldots, m\}$, $\phi^{A_i}(\mathbf{s}_i) = t_i$. It is straightforward to show that if the functions ϕ^{A_i} are differentiable, the functions $V^{\mathbf{A}}$ inherit several properties of the function V. In particular, $V^{\mathbf{A}}$ is increasing and weakly supermodular, $\partial_i V^{\mathbf{A}} > 0$ and $\partial_{ij} V^{\mathbf{A}} \geq 0$.

Let $\{X_j\}_{j\in A_i}$ be the independent random variables describing the information held by bidder i with signals $j \in A_i$. Consider the transformations $\phi^{A_i} : \Delta_i \to \Re$. We will denote by $F_{\phi^{A_i}}$, $f_{\phi^{A_i}}$, and $\overline{F}_{\phi^{A_i}}$ the distribution, density, and survival function of the random variable $\phi^{A_i}\left(\{X_j\}_{j\in A_i}\right)$.

An allocation problem in this context is a collection $\left(n, V, \{X_i\}_{i=1}^n, \{F_i\}_{i=1}^n, \mathbf{A}, \{\phi^{A_i}\}_{A_i\in\mathbf{A}}\right)$, where $\{F_i\}_{i=1}^n$ represent the cumulatives of the n independent random variables $\{X_i\}_{i=1}^n$, $\mathbf{A}$ is an information profile, and $\{\phi^{A_i}\}_{A_i\in\mathbf{A}}$ represent the sufficient scalar statistics. For any allocation problem, define the functions

$$g_i^{\mathbf{A}}(t_i, t_{-i}) = \frac{\overline{F}_{\phi^{A_i}}(t_i)}{f_{\phi^{A_i}}(t_i)} \partial_i V^{\mathbf{A}}(t_i, t_{-i}). \tag{3}$$

We require the following regularity condition for our main result.

Definition An allocation problem is *regular* if, for any information profile $\mathbf{A} = \{A_1, \ldots, A_m\}$, the virtual valuation functions

$$H_i^{\mathbf{A}}(t_i, t_{-i}) = V^{\mathbf{A}}(t_i, t_{-i}) - g_i^{\mathbf{A}}(t_i, t_{-i}) \tag{4}$$

are non-decreasing in t_i for all i.

[2] The differentiability assumption is very mild. By Assumption 1, the functions ϕ^{A_i} are monotone and thus almost everywhere differentiable.

Observe that by Assumption 1, $g_i^{\mathbf{A}}$ is non-decreasing in t_j, for all i and $j \neq i$, and that a sufficient condition for regularity is that $g_i^{\mathbf{A}}$ are non-increasing in t_i. Our regularity condition requiring that virtual valuations are non-decreasing is a generalization of the condition in Myerson (1981) developed by Bulow and Klemperer (1996). We now state our main result.

Theorem 2 *For all regular allocation problems, coarser profiles lead to lower revenues.*

For a sketch of the proof, consider a simple merger where bidder 1 and 2's information is centralized under the control of a new bidder c. Let

$$\mathbf{A} = \{A_1, A_2, \ldots, A_n\}$$

be the pre-merger information profile and

$$\mathbf{A}' = \{A_c, A_3, \ldots, A_n\}$$

represent post-merger, where $A_c = A_1 \cup A_2$.

We show that an incentive-compatible mechanism exists in the pre-merger case that is revenue-equivalent to the optimal mechanism post-merger. Effectively, it treats bidders 1 and 2 *as if* they had merged, even though they do not share information. In the optimal scalar mechanism under $\mathbf{A}'$, each bidder $i \geq 3$ submits a scalar report, t_i, and bidder c submits a scalar report t_c. The proof follows three steps. First, we characterize the optimal post-merger mechanism $\mu^{\mathbf{A}'}$. Second, we construct a mechanism $\mu^{\mathbf{A}}$ for the information profile $\mathbf{A}$, which treats bidders 3 through n identically as $\mu^{\mathbf{A}'}$ but allocates to bidder 2 whenever bidder c would have won under $\mu^{\mathbf{A}}$. Finally, we show that $\mu^{\mathbf{A}}$ is incentive compatible and revenue-equivalent to $\mu^{\mathbf{A}'}$. Since $\mu^{\mathbf{A}}$ is revenue-dominated by the optimal mechanism for information profile $\mathbf{A}$, we establish the desired result.

The central argument behind our revenue comparison in Theorem 2 is quite general. Starting from an optimal mechanism under a concentrated profile, we construct an allocation-equivalent mechanism for a profile where one of the bidding rings is dissolved into its component members. The new mechanism allocates to one representative member, whenever the old mechanism allocated toward the ring, and does not otherwise modify allocations. This generates the same surplus as the old mechanism, but is not necessarily optimal in the new environment. Our argument relies crucially on the independence assumption since this guarantees that the new mechanism satisfies both the individual rationality and incentive compatibility constraints for the representative of the ring.

The requirement that a scalar sufficient statistic exists is also a critical assumption. Our argument assumes the existence of an optimal mechanism under the concentrated profile. While there is no guarantee that such an optimal mechanism exists generically, the scalarization assumption and regularity condition are sufficient for a constructive proof using the approach of Myerson (1981) and Bulow and Klemperer (1996).[3]

[3] Note also that, we represent the transition from concentrated to non-concentrated environments at a very abstract level. Any bidding ring can be dissolved into its component members. This has the flavor of

While we define a regular allocation problem as requiring that the virtual valuation functions, $H_i^{\mathbf{A}}$, are non-decreasing for *every* information profile, it is possible that monotonicity is satisfied only for some information profiles $\mathbf{A}$ and $\mathbf{A}'$. In this case, we still conclude that if $\mathbf{A}'$ is coarser than $\mathbf{A}$, it will be associated with lower revenues. Additionally, if the optimal mechanism under information profile $\mathbf{A}$ requires each buyer to have a strictly positive allocation probability, then a coarser profile leads to *strictly* lower profits. This assures that $\mu^{\mathbf{A}}$ derived from $\mu^{\mathbf{A}'}$ is not optimal, as no buyer is irrelevant.

5 Discussion

Information concentration decreases revenue, even when the auctioneer can respond optimally to the new industry structure. This is similar in spirit to the result of Waehrer and Perry (2003) who find revenues decrease in symmetric *private value* environments even allowing optimal mechanism adjustments that account for less competitive profiles. In common value environments, our result complements that of Bulow and Klemperer (1996), though with one important distinction. Bulow and Klemperer compare an optimal mechanism under a less competitive scenario to a standard auction with more bidders. A no-reserve English auction with $N+1$ bidders revenue dominates any mechanism with N bidders, when signals are independent. This result has been commonly interpreted as stating that the competition effect outweighs any mechanism design variable the auctioneer could introduce. However, N bidders are also collectively less informed than $N+1$ bidders, diluting the argument of the strength of the competitive effect. Our results show that, keeping the informational content constant, we can isolate a competitive effect which reduces the seller's revenues even if we allow the seller full latitude in the choice of mechanisms.

Recently, Mares and Shor (2008) identified a revenue effect of concentration in wallet games, where the value is equal to the sum of the signals, and signals have log-concave density. They consider only symmetric industry profiles and first and second price auctions. For example, six firms each with one independent signal yield higher revenue for the auctioneer than three firms each with two signals. With symmetry, revenue equivalence allows the mechanism design issue to be sidestepped. We can apply our theorem to generalize the result to all information profiles. Consider the following value function:

$$V(X_1, \ldots, X_n,) = \sum X_k$$

where each X_i has log-concave density f_i and distribution F_i. Clearly, the value function is increasing in all of its arguments and is (weakly) supermodular. Further, $\phi^{A_i}(\mathbf{X}_i) = \sum_{k \in A_i} X_k$ is a sufficient statistic for $\mathbf{X}_i$. Thus, we can write the value

Footnote 3 continued
ex-post implementation in its incentive compatibility requirement for the representative of the ring. In multidimensional problems, Bikhchandani (2006) shows that a single-crossing condition akin to scalarization is required for the existence of *ex-post* implementation.

function over summary statistics, $V^{\mathbf{A}}(t_i, t_{-i}) = t_i + \sum_{j \neq i} t_j$. To check our regularity condition, recall that

$$g_i^{\mathbf{A}}(t_i, t_{-i}) = \frac{\overline{F}_{\phi^{A_i}}(t_i)}{f_{\phi^{A_i}}(t_i)} \partial_i V^{\mathbf{A}}(t_i, t_{-i}).$$

Consider X and Y independent variables with log-concave densities f_X and f_Y, which is satisfied by most common distributions (Bagnoli and Bergstrom 2005). This implies that $Z = X + Y$ has a log-concave distribution and survival function (Prékopa 1971, 1973). In particular, $\frac{\overline{F}_Z(t_i)}{f_Z(t_i)}$ is non-increasing. Since $\partial_i V^{\mathbf{A}}(t_i, t_{-i}) = 1$, our regularity condition is satisfied. Therefore, in the wallet game, coarser information profiles lead to lower revenues when signals have log-concave distributions.

Multidimensional signals significantly complicate the analysis of equilibria and call their existence into question (Jackson 2009). To sidestep these complications, researchers have focused on a few specific models for which equilibria have been analytically identified (e.g., Goeree and Offerman 2002; DeBrock and Smith 1983; Mares and Shor 2008). Our results suggest that researchers need not be restricted to these specialized models. Any model that allows for scalar sufficient statistics of each bidder's information can be analyzed in the familiar framework of mechanism design with a scalar message space.[4] While the existence of scalar sufficient statistics is in itself a restrictive assumption, it allows us to examine the impact of information concentration in a more general fashion.

Our results apply to environments in which signals are independent. When signals are affiliated, optimal mechanisms exist that can extract full surplus (Crémer and McLean 1988; McAfee and Reny 1992), thus implying that mergers have no effect on revenue provided at least two bidders remain. These mechanisms have often been criticized as unrealistically sophisticated, though this need not be the case as the following example shows.

Consider four independent random variables Y_i and define $V = \sum_{i=1}^{4} Y_i$. Let the private information of each of four bidders be a signal $X_i = Y_i + Y_{i+1}$ for $i \leq 3$ and $X_4 = Y_4 + Y_1$. In the unmerged case, each bidder receives a scalar signal. Now consider the information profile $A_1 = \{1, 3\}$ and $A_2 = \{2, 4\}$ which yields two perfectly informed bidders and a situation in which every standard auction is full surplus extracting. It is straightforward to see that scalar sufficient statistics exist for these merged entities and that $V = \frac{1}{2} \sum_{i=1}^{4} X_i$, which satisfies our basic assumptions about the value function. Yet, without independence, revenues do not decrease as a result of this concentration. This simple construction exemplifies the need for the independence assumption. Thus, mergers are unlikely to enhance revenue when the auctioneer uses an optimal mechanism: trivially with affiliated signals and strictly with independent signals.

[4] The existence of an optimal scalar mechanism implies that a bidder does not receive information rents for equivalent types—types that share the same value of a sufficient statistic. If this were not the case, then the auctioneer, through sophisticated incentive constraints, would pay a bidder for revealing a specific type among equivalent types (Armstrong and Rochet 1999).

Our results have significant applications for antitrust. In traditional price-setting markets, mergers among smaller, higher-priced firms can reduce average prices by shifting quantity to lower-priced firms post-merger (Werden and Froeb 1994). Similarly, in private value auctions, the auctioneer prefers to face symmetric bidders, holding the average value distribution constant (Cantillon 2008). Even profitable mergers of relatively weak competitors can lead to higher revenues for the auctioneer.[5] This benefit of concentration among smaller market players has been suggested as an efficiency defense in mergers (Dagen and Richards 2006), and several regulators have appeared sensitive to these claims.

Some have conjectured that revenue-increasing mergers are possible in common value auctions as well (e.g., Krishna and Morgan 1997), and specific examples have illustrated pro-competitive merger effects when signals are affiliated, but the auctioneer does not use an optimal mechanism or alter the mechanism across information profiles (DeBrock and Smith 1983; Mares and Shor 2012). Based on these results, some have called for regulatory restraint, advising regulators to err on the side of less oversight when considering common value environments (Froeb and Shor 2005).[6] Klemperer (2005) argues that a positive view of mergers and joint bidding is largely rooted in a misinterpretation of past research and suggests, in line with our results, that it is likely to be harmful for the seller.

Should an auctioneer facing several bidders ever allow a consortium to form or bidders to merge? Perhaps synergies not considered here can have a significant enough positive impact, but the concentration of information, alone, cannot be of benefit to the auctioneer when we allow him to respond optimally via his choice of auction design. Our results describe conditions under which we can identify this unambiguous revenue-reducing impact of information concentration.

Appendix

Proof of Theorem 1

The proof requires three lemmas. Fix the player for whom the value functions admit sufficient statistics to be player 1.

Lemma 1 *Under any incentive compatible mechanism, η, the expected surplus of equivalent types is equal.*

Proof For every mechanism $\eta = (p_i(\cdot), \xi_i(\cdot))$ define

$$\widetilde{V}_j(\mathbf{s}_j; \mathbf{t}_j) = \int V_j(\mathbf{s}_j, \mathbf{s}_{-j}) p_j(\mathbf{t}_j, \mathbf{s}_{-j}) f_{-j}(\mathbf{s}_{-j}) d\mathbf{s}_{-j}$$

[5] Thomas (2004) demonstrates how "a profitable efficiency increasing merger of two relatively small firms creates a stronger competitor that can cause the expected price to fall [in procurement settings], despite the resulting increase in market concentration". (p. 688).

[6] For example, see the comments of Andrew R. Dick, former Acting Chief of the policy section at the DOJ Antitrust Division, J. Mark Gidley, Assistant Attorney General for Antitrust, and David T. Scheffman, former Director of the FTC's Bureau of Competition, who all suggest that asymmetry-reducing mergers can provide market efficiencies (FTC/DOJ 2004).

and

$$\widetilde{\xi}_j(\mathbf{t}_j) = \int \xi_j(\mathbf{t}_j, \mathbf{s}_{-j}) f_{-j}(\mathbf{s}_{-j}) d\mathbf{s}_{-j}$$

Under mechanism η, the expected payoff for player j who has information $\mathbf{s}_j$ and reports $\mathbf{t}_j$ is

$$\widetilde{V}_j(\mathbf{s}_j; \mathbf{t}_j) - \widetilde{\xi}_j(\mathbf{t}_j).$$

The interim incentive compatibility constraint for player 1 is

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) \geq \widetilde{V}_1(\mathbf{s}_1; \mathbf{t}_1) - \widetilde{\xi}_1(\mathbf{t}_1) \tag{5}$$

for all $\mathbf{s}_1$ and $\mathbf{t}_1$. For two equivalent types $\mathbf{s}_1$ and $\mathbf{s}_1'$ and any $\mathbf{t}_1$, we have by definition

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{t}_1) = \widetilde{V}_1(\mathbf{s}_1'; \mathbf{t}_1)$$

and in particular

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) = \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1), \tag{6}$$
$$\widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1') = \widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1'). \tag{7}$$

Combining expressions, we have

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1') = \widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1') \geq \widetilde{\xi}_1(\mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1') \tag{8}$$

where the equality holds by (7) and the inequality by (5), substituting $\mathbf{s}_1'$ for $\mathbf{t}_1$. Also, by substituting $\mathbf{s}_1$ for $\mathbf{t}_1$ and $\mathbf{s}_1'$ for $\mathbf{s}_1$ in (5), we have

$$\widetilde{\xi}_1(\mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1') \geq \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1) - \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1') = \widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1'), \tag{9}$$

where the equality follows from (6). Combining (8) and (9), we obtain

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1') = \widetilde{\xi}_1(\mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1')$$

□

Consider the mechanism η' as defined by Eqs. (1) and (2).

Lemma 2 *If mechanism η is incentive compatible, then η' is an incentive compatible mechanism.*

Proof Define $f_{\phi^{A_1}}$ as the density of $\phi^{A_1}(\mathbf{S}_1)$. Also, for every $\mathbf{s}_1$, such that $\phi^{A_1}(\mathbf{s}_1) = \alpha$, denote by

$$V_i^{A_1}(\alpha; \mathbf{s}_{-1}) = V_i(\mathbf{s}_1, \mathbf{s}_{-1})$$

the parametrization of i's value function based on the sufficient statistic's value. Since η is a mechanism, we have $p_i(\mathbf{s}_1, \mathbf{s}_{-1}) \geq 0$ and $\sum p_i(\mathbf{s}_1, \mathbf{s}_{-1}) \leq 1$ for every i and every $(\mathbf{s}_1, \mathbf{s}_{-1})$. This implies by integration over the set of equivalent types that for i and $\mathbf{s}_1$,

$$\int p_i(\mathbf{t}_1, \mathbf{s}_{-1}) f_1(\mathbf{t}_1|\phi^{A_1}(\mathbf{t}_1) = \phi^{A_1}(\mathbf{s}_1))d\mathbf{t}_1 \geq 0 \Leftrightarrow p_i'(\phi^{A_1}(\mathbf{s}_1), \mathbf{s}_{-1}) \geq 0$$

and similarly

$$\sum p_i'(\phi^{A_1}(\mathbf{s}_1), \mathbf{s}_{-1}) \leq 1.$$

The interim incentive compatibility condition for player j under mechanism η is

$$\widetilde{V}_j(\mathbf{s}_j; \mathbf{s}_j) - \widetilde{\xi}_j(\mathbf{s}_j) \geq \widetilde{V}_j(\mathbf{s}_j; \mathbf{t}_j) - \widetilde{\xi}_j(\mathbf{t}_j)$$

for all $\mathbf{s}_j$ and $\mathbf{t}_j$. For bidder $j \neq 1$,

$$\begin{aligned}
\widetilde{\xi}_j(\mathbf{t}_j) &= \int \xi_j(\mathbf{t}_j, \mathbf{s}_1, \mathbf{s}_{-1j}) f_1(\mathbf{s}_1) f_{-1j}(\mathbf{s}_{-1j}) d\mathbf{s}_{-j} \\
&= \int f_{-1j}(\mathbf{s}_{-1j}) \int f_{\phi^{A_1}}(\alpha) \int \xi_j(\mathbf{t}_j, \mathbf{s}_1, \mathbf{s}_{-1j}) f_1(\mathbf{s}_1|\phi^{A_1}(\mathbf{s}_1) = \alpha) d\mathbf{s}_1 d\alpha d\mathbf{s}_{-1j} \\
&= \int\int \xi_j'(\alpha, \mathbf{t}_j, \mathbf{s}_{-1j}) f_{-1j}(\mathbf{s}_{-1j}) f_{\phi^{A_1}}(\alpha) d\mathbf{s}_{-1j} d\alpha \\
&= \widetilde{\xi}_j'(\mathbf{t}_j).
\end{aligned}$$

Similarly, for all $\mathbf{s}_j$ and $\mathbf{t}_j$,

$$\begin{aligned}
\widetilde{V}_j(\mathbf{s}_j; \mathbf{t}_j) &= \int V_j(\mathbf{s}_j, \mathbf{s}_1, \mathbf{s}_{-1j}) p_j(\mathbf{t}_j, \mathbf{s}_1, \mathbf{s}_{-1j}) f_1(\mathbf{s}_1) f_{-1j}(\mathbf{s}_{-1j}) d\mathbf{s}_{-j} \\
&= \int\int V_j^{A_1}(\alpha; \mathbf{s}_j, \mathbf{s}_{-1j}) p_j'(\alpha, \mathbf{t}_j, \mathbf{s}_{-1j}) f_{-1j}(\mathbf{s}_{-1j}) f_{\phi^{A_1}}(\alpha) d\mathbf{s}_{-1j} d\alpha \\
&= \widetilde{V}_j'(\mathbf{s}_j; \mathbf{t}_j).
\end{aligned}$$

Substituting the identical terms into the incentive compatibility constraint we get

$$\widetilde{V}_j'(\mathbf{s}_j; \mathbf{s}_j) - \widetilde{\xi}_j'(\mathbf{s}_j) \geq \widetilde{V}_j'(\mathbf{s}_j; \mathbf{t}_j) - \widetilde{\xi}_j'(\mathbf{t}_j)$$

for all $\mathbf{s}_j$ and $\mathbf{t}_j$ which indicates that mechanism η' is incentive compatible for player j.

Finally, we need to show that player 1's incentive compatibility constraint is satisfied under mechanism η'. Define

$$\widetilde{V}_1'(\phi^{A_1}(\mathbf{s}_1); \phi^{A_1}(\mathbf{t}_1)) = \int \widetilde{V}_1(\mathbf{s}_1; \mathbf{y}_1) f_1(\mathbf{y}_1|\phi^{A_1}(\mathbf{y}_1) = \phi^{A_1}(\mathbf{t}_1)) d\mathbf{y}_1.$$

and

$$\widetilde{\xi}_1'(\phi^{A_1}(\mathbf{t}_1)) = \int \widetilde{\xi}_1(\mathbf{y}_1) f_1(\mathbf{y}_1 | \phi^{A_1}(\mathbf{y}_1) = \phi^{A_1}(\mathbf{t}_1)) d\mathbf{y}_1$$

which are player 1's expected asset value and expected payment when his type is equivalent to $\mathbf{s}_1$ and he reports a type equivalent to $\mathbf{t}_1$.

The previous lemma establishes that for equivalent types $\mathbf{s}_1$ and $\mathbf{s}_1'$,

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) = \widetilde{V}_1(\mathbf{s}_1'; \mathbf{s}_1') - \widetilde{\xi}_1(\mathbf{s}_1').$$

and therefore, by the definition of equivalent types,

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) = \widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1') - \widetilde{\xi}_1(\mathbf{s}_1').$$

Integrating these relationships along the set of equivalent types with respect to $f_1(\mathbf{s}_1' | \phi^{A_1}(\mathbf{s}_1') = \phi^{A_1}(\mathbf{s}_1))$ results in

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) = \widetilde{V}_1'(\phi^{A_1}(\mathbf{s}_1); \phi^{A_1}(\mathbf{s}_1)) - \widetilde{\xi}_1'(\phi^{A_1}(\mathbf{s}_1)). \tag{10}$$

Player 1's incentive compatibility constraint under η for nonequivalent types $\mathbf{s}_1$ and $\mathbf{t}_1$ is

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) \geq \widetilde{V}_1(\mathbf{s}_1; \mathbf{t}_1) - \widetilde{\xi}_1(\mathbf{t}_1).$$

Integrating these relationships along the set of equivalent types to $\mathbf{t}_1$ yields

$$\widetilde{V}_1(\mathbf{s}_1; \mathbf{s}_1) - \widetilde{\xi}_1(\mathbf{s}_1) \geq \widetilde{V}_1'(\phi^{A_1}(\mathbf{s}_1); \phi^{A_1}(\mathbf{t}_1)) - \widetilde{\xi}_1'(\phi^{A_1}(\mathbf{t}_1)). \tag{11}$$

Combining (10) and (11) yields

$$\widetilde{V}_1'(\phi^{A_1}(\mathbf{s}_1); \phi^{A_1}(\mathbf{s}_1)) - \widetilde{\xi}_1'(\phi^{A_1}(\mathbf{s}_1)) \geq \widetilde{V}_1'(\phi^{A_1}(\mathbf{s}_1); \phi^{A_1}(\mathbf{t}_1)) - \widetilde{\xi}_1'(\phi^{A_1}(\mathbf{t}_1)).$$

which is the incentive compatibility constraint for player 1 under mechanism η'. □

Lemma 3 *The mechanisms η and η' are revenue-equivalent.*

Proof The expected revenue for the seller under mechanism η is

$$\begin{aligned} ER(\eta) &= \int \sum_k \xi_k(\mathbf{s}) f(\mathbf{s}) d\mathbf{s} \\ &= \sum_k \int \xi_k(\mathbf{s}) \prod f_i(\mathbf{s}_i) d\mathbf{s}_i \\ &= \sum_k \int f_{-1}(\mathbf{s}_{-1}) \int f_{\phi^{A_1}}(\alpha) \int \xi_k(\mathbf{s}_1, \mathbf{s}_{-1}) f_1(\mathbf{s}_1 | \phi^{A_1}(\mathbf{s}_1) = \alpha) d\mathbf{s}_1 d\alpha d\mathbf{s}_{-1}. \end{aligned}$$

$$
\begin{aligned}
&= \sum_k \int \int \xi_k'(\alpha, \mathbf{s}_{-i}) f_{-1}(\mathbf{s}_{-1}) f_{\phi^{A_1}}(\alpha) d\alpha d\mathbf{s}_{-1} \\
&= ER(\eta')
\end{aligned}
$$

□

Proof of Theorem 1 Follows from the above three lemmas. □

Proof of Theorem 2

Notation

For notational convenience, we consider the case where buyer 1 and 2's information is centralized under the control of a new buyer c. Let

$$\mathbf{A} = \{A_1, A_2, A_3, \ldots, A_m\}$$

and

$$\mathbf{A}' = \left\{A_c, A_3', \ldots, A_m'\right\}$$

where $A_c = A_1 \cup A_2$ and $A_i = A_i'$ for all $i \geq 3$. All the other cases can be derived from this analysis. For simplicity, we drop the superscript in $\phi^{A_c}(X_1, X_2)$ and denote the sufficient statistic by $\phi(X_1, X_2)$.

For every t in the support of $\phi(X_1, X_2)$ define:

$$\underline{x}_1(t) = \inf\left\{x_1 \mid \exists x_2 \in [\underline{z}_2, \overline{z}_2], \phi(x_1, x_2) = t\right\} \tag{12}$$

and

$$\overline{x}_1(t) = \sup\left\{x_1 \mid \exists x_2 \in [\underline{z}_2, \overline{z}_2], \phi(x_1, x_2) = t\right\}, \tag{13}$$

analogously define $\underline{x}_2(t)$ and $\overline{x}_2(t)$. Note that these objects are well-defined and increasing when $\phi(\cdot, \cdot)$ is increasing.

Define implicitly for every t, $\psi_2(\cdot, t) : [\underline{x}_1(t), \overline{x}_1(t)] \to [\underline{x}_2(t), \overline{x}_2(t)]$

$$\phi(x_1, \psi_2(x_1, t)) = t$$

and $\psi_1(\cdot, t) : [\underline{x}_2(t), \overline{x}_2(t)] \to [\underline{x}_1(t), \overline{x}_1(t)]$

$$\phi(\psi_1(x_2, t), x_2) = t.$$

Note that $\psi_1(\cdot, t)$ and $\psi_2(\cdot, t)$ are well-defined decreasing functions and $\psi_1^{-1}(\cdot, t) = \psi_2(\cdot, t)$. Further, when ϕ is differentiable then so are ψ and ψ_2, and

$$\frac{d}{dx_1}\phi(x_1, \psi_2(x_1, t)) = 0 \Leftrightarrow \partial_1\phi(x_1, \psi_2(x_1, t)) = -\partial_2\phi(x_1, \psi_2(x_1, t))\partial_{x_1}\psi_2(x_1, t)$$

or

$$\partial_{x_1}\psi_2(x_1,t) = -\frac{\partial_1\phi(x_1,\psi_2(x_1,t))}{\partial_2\phi(x_1,\psi_2(x_1,t))}$$

and analogously

$$\partial_{x_2}\psi_1(x_2,t) = -\frac{\partial_2\phi(\psi_1(x_2,t),x_2)}{\partial_1\phi(\psi_1(x_2,t),x_2)}.$$

Note that if $\phi(x_1,x_2)=t$

$$V^{\mathbf{A}'}(t,t_{-12}) = V^{\mathbf{A}}(x_1,\psi_2(x_1,t),t_{-12}) \tag{14}$$

and therefore

$$\begin{aligned}\partial_t V^{\mathbf{A}'}(t,t_{-12}) &= \partial_2 V^{\mathbf{A}}(x_1,\psi_2(x_1,t),t_{-12})\partial_t\psi_2(x_1,t)\\ &= \frac{\partial_2 V^{\mathbf{A}}(x_1,\psi_2(x_1,t),t_{-12})}{\partial_2\phi\,(x_1,\psi_2(x_1,t))}.\end{aligned} \tag{15}$$

By a similar argument one can show that

$$\partial_t V^{\mathbf{A}'}(t,t_{-12}) = \frac{\partial_1 V^{\mathbf{A}}(\psi_1(x_2,t),x_2,t_{-12})}{\partial_1\phi\,(\psi_1(x_2,t),x_2)}.$$

and for all $j\geq 3$

$$\partial_{tj} V^{\mathbf{A}'}(t,t_{-1}) = \frac{\partial_{1j} V^{\mathbf{A}}(\psi_1(x_2,t),x_2,t_{-12})}{\partial_1\phi\,(\psi_1(x_2,t),x_2)},$$

which means that $\partial_{tj}V^{\mathbf{A}'}$ and $\partial_{1j}V^{\mathbf{A}}$ will have the same sign.

Lemma 4 *The survival function and density function of $\phi(X_1,X_2)$ are given by*

$$\overline{F}_\phi(t) = \int_{-\infty}^{\infty} f_1(x)\overline{F}_2(\psi_2(x,t))dx = \int_{-\infty}^{\infty} f_2(x)\overline{F}_1(\psi_1(x,t))dx, \textit{ and} \tag{16}$$

$$f_\phi(t) = \int_{-\infty}^{\infty} \frac{f_1(x)f_2(\psi_2(x,t))}{\partial_2\phi(x,\psi_2(x,t))}dx = \int_{-\infty}^{\infty} \frac{f_2(x)f_1(\psi_1(x,t))}{\partial_1\phi(\psi_1(x,t),x)}dx. \tag{17}$$

Proof Note that

$$\Pr\left[\phi(X_1,X_2)\geq t\right] = \overline{F}_\phi(t) = \int \Pr\left[\phi(X_1,X_2)\geq t|X_1=x\right] f_1(x)dx.$$

Since X_1 and X_2 are independent, we have

$$\Pr[\phi(X_1, X_2) \geq t | X_1 = x] = \Pr[X_2 \geq \psi_2(x,t) | X_1 = x] = \overline{F}_2(\psi_2(x,t)).$$

Substituting back into the integral yields the first expression. Note that

$$\frac{d}{dt}\phi(x, \psi_2(x,t)) = 1 \Leftrightarrow \partial_2\phi(x, \psi_2(x,t))\partial_t\psi_2(x,t) = 1$$

or

$$\partial_t\psi_2(x,t) = \frac{1}{\partial_2\phi(x, \psi_2(x,t))}. \tag{18}$$

Then,

$$\begin{aligned} f_\phi(t) &= -\partial_t\overline{F}_\phi(t) \\ &= \int_{-\infty}^{\infty} f_1(x) f_2(\psi_2(x,t))\partial_t\psi_2(x,t)dx \end{aligned}$$

Substituting Eq. (18) yields:

$$= \int_{-\infty}^{\infty} \frac{f_1(x) f_2(\psi_2(x,t))}{\partial_2\phi(x, \psi_2(x,t))}dx$$

□

Proof of Theorem 2 Under our assumptions, every buyer's information can be summarized by $T_i = \phi^{A_i}(\mathbf{S}_i)$ and in any scalar mechanism for information profile $\mathbf{A}$ only reports t_i are required from buyers. In the concentrated environment $\mathbf{A}'$, a scalar mechanism will require reports $t_i = T_i$ from buyers 3 through m, while it will ask buyer c to submit a report $t_c = \phi(t_1, t_2)$. Denote by $f_i(t_i)$, $F_i(t_i)$ and $[\underline{a}_i, \overline{a}_i]$ the density, distribution function and, respectively, support of the random variables T_i. Further, denote by f_ϕ and F_ϕ the density and distribution function of the random variable $\phi(T_1, T_2)$. Denote by t_{-i} and t_{-ij} the vector of reports excluding buyer i or buyers i and j.

Consider the virtual valuation of player c,

$$H_c^{\mathbf{A}'}(t_c, t_{-12}) = V^{\mathbf{A}'}(t_c, t_{-12}) - \frac{\overline{F}_\phi(t_c)}{f_\phi(t_c)}\partial_{t_c}V^{\mathbf{A}'}(t_c, t_{-12}).$$

Under the regularity assumption, $H_c^{\mathbf{A}'}(\cdot, t_{-12})$ is non-decreasing.

Following (Bulow and Klemperer 1996), for any mechanism $\eta' = \left(p_i', \xi_i'\right)_{i\in\{c,3,\ldots,m\}}$ in environment $\mathbf{A}'$, the seller's revenue is

$$R(\eta') = \int \left(\sum_{i \geq 3} p'_i(t_i, t_{-i}) H_i^{\mathbf{A}'}(t_i, t_{-i}) + p'_c(t_c, t_{-12}) H_c^{\mathbf{A}'}(t_c, t_{-12}) + p'_0 v_0 \right)$$
$$\times f_\phi(t_c) f(t_{-12}) dt_c dt_{-12},$$

where v_0 is the seller's reservation value. Point-by-point maximization of the integrand yields the optimal solution $\mu^{\mathbf{A}'}$ where

$$p'_i(t_i, t_{-i}) = 1 \Leftrightarrow H_i^{\mathbf{A}'}(t_i, t_{-i}) \geq \max_{j \neq i}(v_0, H_c^{\mathbf{A}'}(t_c, t_{-12}), H_j^{\mathbf{A}'}(t_j, t_{-j}))$$

and zero otherwise. Since the functions $H_i^{\mathbf{A}'}$ are non-decreasing, $H_i^{\mathbf{A}'}(t_i, t_{-i}) \geq v_0$ implies $H_i^{\mathbf{A}'}(t'_i, t_{-i}) \geq v_0$ for all $t'_i \geq t_i$. Further, for all j, and for all $t'_i \geq t_i$, by Assumption 1 we have $H_i^{\mathbf{A}'}(t'_i, t_{-i}) \geq H_j^{\mathbf{A}'}(t'_i, t_{-i})$. In particular, for bidder c we have

$$p'_c(t_c, t_{-12}) = 1 \Leftrightarrow H_c^{\mathbf{A}'}(t_c, t_{-12}) \geq \max_i(v_0, H_i^{\mathbf{A}'}(t_i, t_{-i}))$$

and zero otherwise. We conclude thus that for any t_{-12} and v_0, the set of types for which bidder c gets the object is given by

$$M_c = \{(t_1, t_2) | \phi(t_1, t_2) = t_c \geq \tau(t_{-12}, v_0)\}.$$

for some function, τ. The expected payment received by the auctioneer from bidder c is therefore,

$$\xi_c^{\mathbf{A}'} = \int \left(\int_{t_c > \tau(t_{-12}, v_0)} H_c^{\mathbf{A}'}(t_c, t_{-12}) f_\phi(t_c) dt_c \right) f(t_{-12}) dt_{-12}.$$

Define

$$Q'(t) = \int_{t_c > t} H_c^{\mathbf{A}'}(t_c, t_{-12}) f_\phi(t_c) dt_c,$$

then

$$\xi_c^{\mathbf{A}'} = \int Q'(\tau(t_{-12}, v_0)) f(t_{-12}) dt_{-12}.$$

Consider a mechanism $\mu^{\mathbf{A}} = (p_i, \xi_i)_{i \in \{1,2,\ldots,m\}}$ for the environment $\mathbf{A}$, with the following properties $p_1(t_1, t_{-1}) \equiv 0$, $p_2(t_1, t_2, t_{-12}) \equiv p'_c(\phi(t_1, t_2), t_{-12})$ and $p_i(t_i, t_{-i}) \equiv p'_i(t_i, t_{-i})$ for all $i \geq 3$. The mechanism $\mu^{\mathbf{A}}$ is incentive compatible since all the functions are non-decreasing in own type.[7] Furthermore, the expected payment received from bidders 3 through n ,under $\mu^{\mathbf{A}}$ and $\mu^{\mathbf{A}'}$ are the same.

The expected payment from bidder 2 in this case is therefore

$$\xi_2^{\mathbf{A}} = \int \left(\iint_{M_c} H_2^{\mathbf{A}}(t_1, t_2, t_{-12}) f_1(t_1) f_2(t_2) dt_1 dt_2 \right) f(t_{-12}) dt_{-12}.$$

Define

$$Q(t) = \iint_{\phi(t_1,t_2)\geq t} H_2^{\mathbf{A}}(t_1, t_2, t_{-12}) f_1(t_1) f_2(t_2) dt_1 dt_2,$$

then

$$\xi_2^{\mathbf{A}} = \int (Q(\tau(t_{-12}, v_0))) f(t_{-12}) dt_{-12}.$$

In particular, we will show that for all t

$$Q'(t) = Q(t)$$

the expected payments of bidder c under $\mu^{\mathbf{A}'}$ and those of bidder 2 under $\mu^{\mathbf{A}}$ coincide, which makes the two mechanisms revenue-equivalent. Define

$$\underline{t}_1(t) = \inf\{t_1 | \exists t_2, \phi(t_1, t_2) = t\} \text{ and } \overline{t}_1(t) = \sup\{t_1 | \exists t_2, \phi(t_1, t_2) = t\}, \quad (19)$$

and note that $Q(t)$ may be expressed as

$$Q(t) = \iint_{\phi(t_1,t_2)\geq t} \left(V^{\mathbf{A}}(t_1, t_2, t_{-12}) - \frac{\overline{F}_2(t_2)}{f_2(t_2)} \partial_2 V^{\mathbf{A}}(t_1, t_2, t_{-12}) \right) f_1(t_1) f_2(t_2) dt_1 dt_2 \quad (20)$$

We consider each component separately. First,

$$\begin{aligned} &\iint_{\phi(t_1,t_2)\geq t} V^{\mathbf{A}}(t_1, t_2, t_{-12}) f_1(t_1) f_2(t_2) dt_1 dt_2 \\ &\quad = \int_{\underline{t}_1(t)}^{\overline{t}_1(t)} \int_{t_2 \geq \psi_2(t_1,t)} V^{\mathbf{A}}(t_1, t_2, t_{-12}) f_1(t_1) f_2(t_2) dt_2 dt_1 \end{aligned}$$

[7] Bidder 1's payment will be zero.

Fix t_1 and introduce the change in variable $t_2 = \psi_2(t_1, t_c)$,observing that $dt_2 = \partial_{t_c}\psi_2(t_1, t_c)dt_c$.

The integral becomes

$$\int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \int_{t_c \geq t} V^{\mathbf{A}}(t_1, \psi_2(t_1, t_c), t_{-12}) f_1(t_1) f_2(\psi_2(t_1, t_c)) \partial_{t_c}\psi_2(t_1, t_c) dt_c dt_1$$
$$= \int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \int_{t_c \geq t} \frac{V^{\mathbf{A}}(t_1, \psi_2(t_1, t_c), t_{-12}) f_1(t_1) f_2(\psi_2(t_1, t_c))}{\partial_2\phi(t_1, \psi_2(t_1, t_c))} dt_c dt_1$$
$$= \int_{t_c \geq t} \int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \frac{V^{\mathbf{A}'}(t_c, t_{-12}) f_1(t_1) f_2(\psi_2(t_1, t_c))}{\partial_2\phi(t_1, \psi_2(t_1, t_c))} dt_1 dt_c$$
$$= \int_{t_c \geq t} V^{\mathbf{A}'}(t_c, t_{-12}) \int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \frac{f_1(t_1) f_2(\psi_2(t_1, t_c))}{\partial_2\phi(t_1, \psi_2(t_1, t_c))} dt_1 dt_c$$
$$= \int_{t_c \geq t} V^{\mathbf{A}'}(t_c, t_{-12}) f_\phi(t) dt_c.$$

Where the first equality follows from Eq. (18), the second from (14) and Fubini's theorem, and the last by Lemma 4.

The second part of the integral in (20) is

$$-\iint_{\phi(t_1,t_2)\geq t} \left(\frac{\overline{F}_2(t_2)}{f_2(t_2)} \partial_2 V^{\mathbf{A}}(t_1, t_2, t_{-12})\right) f_1(t_1) f_2(t_2) dt_1 dt_2$$
$$= -\iint_{\phi(t_1,t_2)\geq t} f_1(t_1)\partial_2 V^{\mathbf{A}}(t_1, t_2, t_{-12})\overline{F}_2(t_2) dt_1 dt_2$$
$$= -\int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \int_{t_2 \geq \psi_2(t_1,t)} f_1(t_1)\partial_2 V^{\mathbf{A}}(t_1, t_2, t_{-12})\overline{F}_2(t_2) dt_2 dt_1$$
$$= -\int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \int_{t_c \geq t} f_1(t_1)\partial_2 V^{\mathbf{A}}(t_1, \psi_2(t_1, t_c), t_{-12})\overline{F}_2(\psi_2(t_1, t_c))\partial_{t_c}\psi_2(t_1, t_c) dt_c dt_1$$
$$= -\int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \int_{t_c \geq t} \frac{\partial_2 V^{\mathbf{A}}(t_1, \psi_2(t_1, t_c), t_{-12})}{\partial_2\phi(t_1, \psi_2(t_1, t_c))} f_1(t_1)\overline{F}_2(\psi_2(t_1, t_c)) dt_c dt_1$$

$$= -\int_{t_c \geq t} \int_{\underline{t}_1(t)}^{\bar{t}_1(t)} \partial_{t_c} V^{\mathbf{A}'}(t_c, t_{-12}) f_1(t_1) \overline{F}_2(\psi_2(t_1, t_c)) dt_1 dt_c$$

$$= -\int_{t_c \geq t} \partial_{t_c} V^{\mathbf{A}'}(t_c, t_{-12}) \int_{\underline{t}_1(t)}^{\bar{t}_1(t)} f_1(t_1) \overline{F}_2(\psi_2(t_1, t_c)) dt_1 dt_c$$

$$= -\int_{t_c \geq t} \partial_{t_c} V^{\mathbf{A}'}(t_c, t_{-12}) \overline{F}_\phi(t_c) dt_c$$

Combining the two results we have

$$Q(t) = \int_{t_c \geq t} \left(V^{\mathbf{A}'}(t_c, t_{-12}) f_\phi(t_c) - \partial_{t_c} V^{\mathbf{A}'}(t_c, t_{-12}) \overline{F}_\phi(t_c) \right) dt_c$$

$$= \int_{t_c \geq t} \left(V^{\mathbf{A}'}(t_c, t_{-12}) - \frac{\overline{F}_\phi(t_c)}{f_\phi(t_c)} \partial_{t_c} V^{\mathbf{A}'}(t_c, t_{-12}) \right) f_\phi(t_c) dt_c$$

$$= \int_{t_c > t} H_c^{\mathbf{A}'}(t_c, t_{-12}) f_\phi(t_c) dt_c$$

$$= Q'(t)$$

which means that $\mu^{\mathbf{A}}$ and $\mu^{\mathbf{A}'}$ generate the same expected revenue. However, under $\mu^{\mathbf{A}}$, buyer 1 receives the good with probability zero. If the optimal mechanism in environment **A** allocates to buyer 1 with strictly positive probability, then it is, by definition, revenue superior to $\mu^{\mathbf{A}}$ and hence to $\mu^{\mathbf{A}'}$. $\square$

References

Armstrong M, Rochet J-C (1999) Multi-dimensional screening: a user's guide. Eur Econ Rev 43(4–6):959–979

Athey S, Levin J (2001) The value of information in monotone decision problems. Working Paper, MIT

Bagnoli M, Bergstrom T (2005) Log-concave probability and its applications. Econ Theory 26(2):445–469

Bergemann D, Valimaki J (2002) Information acquisition and efficient mechanism design. Econometrica 70(3):1007–1033

Biais B, Martimort D, Rochet J-C (2000) Competing mechanisms in a common value environment. Econometrica 68(4):799–837

Bikhchandani S, Riley J (1991) Equilibria in open common value auctions. J Econ Theory 53(1):101–130

Bikhchandani S (2006) Ex post implementation in environments with private goods. Theor Econ 1(3):369–393

Bulow J, Klemperer P (1996) Auctions versus negotiations. Am Econ Rev 86(1):180–194

Bulow J, Klemperer P (2002) Prices and the winner's curse. RAND J Econ 33(1):1–21

Cantillon E (2008) The effect of bidders' asymmetries on expected revenue in auctions. Games Econ Behav 62(1):1–25

Crémer J, McLean R (1988) Full extraction of the surplus in bayesian and dominant strategy auctions. Econometrica 56(6):1247–1257

Dagen R, Richards D (2006) Merger theory and evidence: the baby-food case reconsidered. Tufts University Department of Economics, Working Paper
Dalkir S, Logan JW, Masson RT (2000) Mergers in symmetric and asymmetric noncooperative auction markets: the effects on prices and efficiency. Int J Ind Organ 18(3):383–413
DeBrock L, Smith J (1983) Joint bidding, information pooling, and the performance of petroleum lease auctions. Bell J Econ 14(2):395–404
Froeb L, Shor M (2005) Auction models. In: Harkider JD (eds) Econometrics: legal, practical, and technical issues, pp 225–246. American Bar Association Section of Antitrust Law
FTC/DOJ (2004) Joint workshop on merger enforcement. February 17–19, Washington, D.C.
Goeree JK, Offerman T (2002) Efficiency in auctions with private and common values: an experimental study. Am Econ Rev 93(3):625–643
Jackson MO (2009) Non-existence of equilibrium in vickrey, second-price, and english auctions. Rev Econ Des 13(1):137–145
Klemperer P (2005) Bidding markets. UK Competition Commission, Occasional Paper No 1
Klemperer P (1998) Auctions with almost common values: the 'wallet game' and its applications. Eur Econ Rev 42(3–5):757–769
Krishna V, Morgan J (1997) (anti-) competitive effects of joint bidding and bidder restrictions. Penn State University and Princeton University, Working Paper
Mares V, Harstad RM (2003) Private information revelation in common-value auctions. J Econ Theory 109(2):264–282
Mares V, Shor M (2008) Industry concentration in common value auctions. Econ Theory 35:37–56
Mares V, Shor M (2012) On the competitive effects of bidding syndicates. B.E. J Econ Anal Policy [Frontiers] 12:1–32
Matthews SA (1984) Information acquisition in discriminatory auctions. In: Boyer M, Kihlstrom RE (eds) Bayesian models in economic theory. Elsevier Science, pp 181–207
McAfee RP, Reny PJ (1992) The competitive effects of mergers between asymmetric firms. Correl Inf Mech Des 60(2):395–421
Myerson R (1981) Optimal auction design. Math Oper Res 6(1):58–73
Persico N (2000) Information acquisition in auctions. Econometrica 68(1):135–148
Prékopa A (1971) Logarithmic concave measures with application to stochastic programming. ACTA Sci Math (Szeged) 32:301–316
Prékopa A (1973) On logarithmic concave measures and functions. ACTA Sci Math (Szeged) 34:335–343
Reny PJ, Zamir S (2004) On the existence of pure strategy monotone equilibria in asymmetric first-price auctions. Econometrica 72(4):1105–1125
Thomas CJ (2004) The competitive effects of mergers between asymmetric firms. Int J Ind Organ 22(5):679–692
Tschantz S, Crooke P, Froeb L (2000) Mergers in sealed vs. oral auctions. Int J Econ Bus 7(2):201–213
Waehrer K, Perry MK (2003) The effects of mergers in open-auction markets. RAND J Econ 34(2):287–304
Werden GJ, Froeb LM (1994) The effects of mergers in differentiated products industries: logit demand and merger policy. J Law Econ Organ 10(2):407–426